§6.4: The 100% Rule

1.] Giapetto's Workshop: Suppose x_1 and x_2 are the number of soldiers and trains, respectively, that Giapetto's produces and sells from his workshop. He sells each toy for a profit but is limited by two types of skilled labor hours: finishing (constraint 1) and carpentry (constraint 2). The third constraint is a demand constraint. The LP is below along with the optimal tableau.

Maximize Profit: $z = 3x_1 + 2x_2$

Subject to:
$$2x_1 + x_2 \leq 100$$
$$x_1 + x_2 \leq 80$$
$$x_1 \leq 40$$
$$x_1, x_2 \geq 0$$

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
B4	Values: x1	20	0	$c_1 =$ 3	$I_1 =$ 1	$D_1 =$ 1
C4	Values: x2	60	0	$c_2 =$ 2	$I_2 =$ 1	$D_2 =$ 0.5

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
D10	Finishign Totals:	100	1	$b_1 =$ 100	$I_1 =$ 20	$D_1 =$ 20
D11	Carpentry Totals:	80	1	$b_2 =$ 80	$I_2 =$ 20	$D_2 =$ 20
D12	Demand Totals:	20	0	$b_3 =$ 40	$I_3 =$ 1E+30	$D_3 =$ 20

a.) Suppose the price of selling soldiers is increased to \$3.50 and the price of trains is decreased to \$1.80. With these changes implemented, will the current set of optimal decision variables remain optimal?

From the Excel output, we have the following allowable increases/decreases:

$D_1 = 1$, $I_1 = 1$

$D_2 = .50$, $I_2 = 1$

If $c_1 = 3.50$, then $\Delta c_1 = .50$.

If $c_2 = 1.80$, then $\Delta c_2 = -.20$.

Thus, we define

$$r_1 = \frac{\Delta c_1}{I_1} = \frac{.50}{1} = .50$$

$$r_2 = -\frac{\Delta c_2}{D_2} = -\frac{(-.20)}{.50} = .40$$

Hence, $r_1 + r_2 = .5 + .4 = .9 \leq 1$.

The basis remains optimal.

b.) Suppose the finishing hours are decreased to 85 and the carpentry hours are increased to 95. Will the current set of optimal decision variables remain optimal?

From the Excel output, we have the following allowable increases/decreases:

$D_1 = 20$, $I_1 = 20$

$D_2 = 20$, $I_2 = 20$

If $b_1 = 85$, then $\Delta b_1 = -15$.

If $b_2 = 95$, then $\Delta b_2 = 15$.

Thus, we define

$$r_1 = -\frac{\Delta b_1}{D_1} = -\frac{(-15)}{20} = .75$$

$$r_2 = \frac{\Delta b_2}{I_2} = \frac{15}{20} = .75$$

Hence, $r_1 + r_2 = .75 + .75 = 1.5 > 1$.

The basis may or may not remain optimal — inconclusive.