

§6.3 (PART 2): SENSITIVITY ANALYSIS

- 1.] GIAPETTO'S WORKSHOP: Suppose x_1 and x_2 are the number of soldiers and trains, respectively, that Giapetto's produces and sells from his workshop. He sells each toy for a profit but is limited by two types of skilled labor hours: finishing (constraint 1) and carpentry (constraint 2). The third constraint is a demand constraint. The LP is below along with the optimal tableau.

Maximize Profit: $z = 3x_1 + 2x_2$

Subject to: $2x_1 + x_2 \leq 100$

$x_1 + x_2 \leq 80$

$x_1 \leq 40$

$x_1, x_2 \geq 0$

| Row | Basic | z | x_1 | x_2 | s_1 | s_2 | s_3 | RHS |
|-----|-------|-----|-------|-------|-------|-------|-------|-----|
| 0 | z | 1 | 0 | 0 | 1 | 1 | 0 | 180 |
| 1 | x_1 | 0 | 1 | 0 | 1 | -1 | 0 | 20 |
| 2 | x_2 | 0 | 0 | 1 | -1 | 2 | 0 | 60 |
| 3 | s_3 | 0 | 0 | 0 | -1 | 1 | 1 | 20 |

Suppose that Giapetto is considering manufacturing toy boats. A toy boat uses 2 carpentry hours and 1 finishing hour and the demand is unlimited.

- a.) Reformulate the LP in standard form.

$$\text{Maximize } z = 3x_1 + 2x_2 + 3.50x_3$$

$$\begin{aligned} \text{Subject to } 2x_1 + x_2 + x_3 + s_1 &= 180 \\ x_1 + x_2 + 2x_3 + s_2 &= 80 \\ x_1 + s_3 &= 40 \end{aligned} \quad \vec{a}_3 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, C_3 = 3.50$$

- b.) If a toy boat contributes \$3.50 to profit, should Giapetto manufacture any toy boats?

Price out the new activity by computing \bar{c}_3 under the current optimal basis:

$$\begin{aligned} \bar{c}_3 &= \vec{c}_B^T B^{-1} \vec{a}_3 - c_3 \\ &= [3 \ 2 \ 0] \begin{bmatrix} -1 & -1 & 0 \\ -1 & 2 & 0 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} - 3.50 \\ &= [1 \ 1 \ 0] \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} - 3.50 \\ &= 3 - 3.50 \\ &= -0.50 < 0 \end{aligned}$$

At this price of \$3.50 per toy boat, it would be beneficial to manufacture toy boats.