§4.7: Alternative Optimal Solutions

1.] Find two optimal solutions to the two-variable LP problem using the Simplex Method:

$$\text{Maximize:} \quad z = 4x_1 + 2x_2$$

$$\begin{aligned} \text{Subject to:} \quad 2x_1 + x_2 &\leq 100 \\ x_1 + x_2 &\leq 80 \\ x_1 \quad\quad &\leq 40 \\ x_1, x_2 &\geq 0 \end{aligned}$$

Row	Basic		RHS
0	z		
1			
2			
3			

Row	Basic		RHS
0'	z		
1'			
2'			
3'			

Row	Basic		RHS
0"	z		
1"			
2"			
3"			

Row	Basic		RHS
0"'	z		
1"'			
2"'			
3"'			

2.] In the previous example, denote $\boldsymbol{b}_1$ and $\boldsymbol{b}_2$ as the two optimal solutions. Show that $\boldsymbol{x} = \sigma_1 \boldsymbol{b_1} + \sigma_2 \boldsymbol{b}_2$, with $\sigma_1 + \sigma_2 = 1$, is also optimal.

3.] Consider an LP with objective function $z = \boldsymbol{c}^T \boldsymbol{x}$ and feasible region given by $A\boldsymbol{x} = \boldsymbol{b}$ with $\boldsymbol{x} \geq \boldsymbol{0}$. Suppose $\boldsymbol{b}_i$ for $i = 1, 2, \ldots, k$ are all optimal solutions. Show that any convex combination of these optimal solutions is also optimal.

4.] Below are two partial tableaus for two separate maximization LPs. Are they optimal? What are the basic and nonbasic variables of each problem? Which LP has alternative optima?

Row	z	x_1	x_2	s_1	s_2	RHS
0	1	0	0	2	3	10
1	0	1	0	3	2	4
2	0	0	1	1	1	3

Row	z	x_1	x_2	s_1	s_2	RHS
0	1	0	0	0	2	2
1	0	1	0	-1	1	2
2	0	0	1	-2	3	3