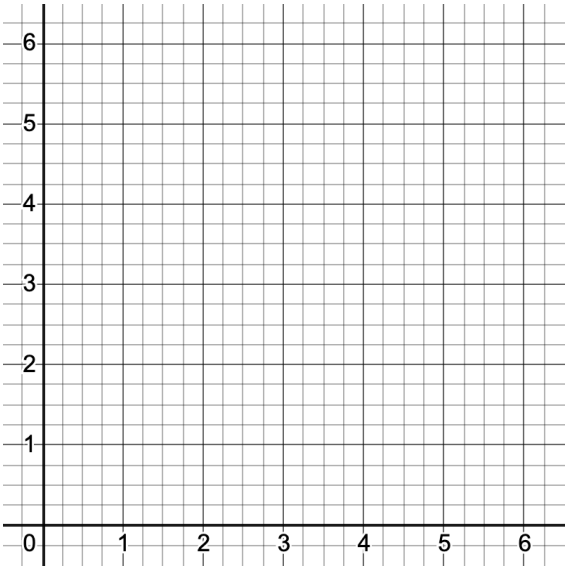
§4.11 (part 1): Degeneracy and Convergence of Simplex Method

1.] Consider the following two variable LP below:

$$\begin{aligned} \text{Maximize:} \quad & z = 5x_1 + 2x_2 \\ \text{Subject to:} \quad & x_1 + x_2 \leq 6 \\ & x_1 - x_2 \leq 0 \\ & x_1, x_2 \geq 0 \end{aligned}$$

Sketch the feasible space on the graph below, labeling the corner points of the feasible space. Convert the LP to standard form and identify all basic solutions with their corresponding objective values by filling out the table below.



Nonbasic Variables	Basic Variables	Basic Solution	Corner Point	Feasible?	Obj Value (z)
x_1, x_2					
x_1, s_1					
x_1, s_2					
x_2, s_1					
x_2, s_2					
s_1, s_2					

2.] Show that even though the initial tableau is not degenerate, later iterations may exhibit degeneracy.

$$\text{Maximize:} \quad z = 5x_1 + 3x_2$$

$$\begin{aligned} \text{Subject to:} \quad 4x_1 + 2x_2 &\leq 12 \\ 4x_1 + x_2 &\leq 10 \\ x_1 + x_2 &\leq 4 \\ x_1, x_2 &\geq 0 \end{aligned}$$

Row	Basic	z	x_1	x_2	s_1	s_2	s_3	RHS
0	z	1	−5	−3	0	0	0	0
1	s_1	0	4	2	1	0	0	12
2	s_2	0	4	1	0	1	0	10
3	s_3	0	1	1	0	0	1	4

Row	Basic	z	x_1	x_2	s_1	s_2	s_3	RHS
0’	z							
1’								
2’								
3’								

Row	Basic	z	x_1	x_2	s_1	s_2	s_3	RHS
0”	z							
1”								
2”								
3”								

Row	Basic	z	x_1	x_2	s_1	s_2	s_3	RHS
0”’	z							
1”’								
2”’								
3”’								