

§4.3: DIRECTION OF UNBOUNDEDNESS

- 1.] DORIAN AUTO: In Example 2 of Chapter 3, the text formulates the LP for Dorian Auto manufacturers. The LP is given below in standard form:

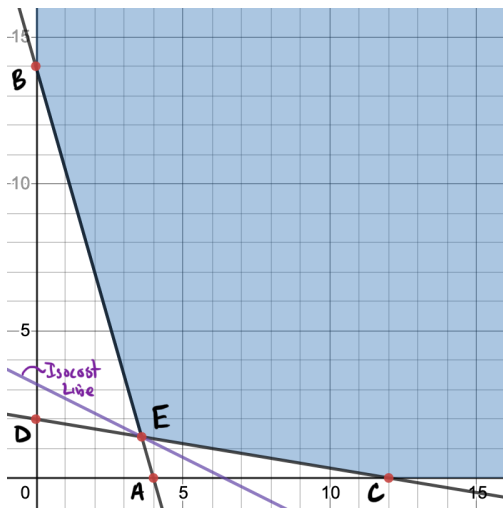
$$\text{Minimize: } z = 50x_1 + 100x_2$$

$$\text{Subject to: } 7x_1 + 2x_2 - e_1 = 28$$

$$2x_1 + 12x_2 - e_2 = 24$$

$$x_1, x_2, e_1, e_2 \geq 0$$

Sketching the feasible region in two-dimensional space gives the following unbounded convex set:



- a.) Show that the feasible solution corresponding to the point $(6, 7)$, call it \mathbf{x} , can be rewritten as a linear combination of the form $\mathbf{x} = \sigma_1 \mathbf{b}_1 + \sigma_2 \mathbf{b}_2$, where \mathbf{b}_1 and \mathbf{b}_2 correspond to the basic feasible solutions at the points $(0, 14)$ and $(12, 0)$, respectively, and $\sigma_1 + \sigma_2 = 1$.

- 2.] Show that the basic feasible solution with non-basic variables $e_1, e_2 = 0$ corresponds to corner point E by solving a particular two-dimensional linear system.

- 3.] Consider the point $(14, 4)$ in the feasible space. This point is not a basic feasible solution, why? Determine the vector \mathbf{x} that corresponds to this point. Represent this point as a combination of the form $\mathbf{x} = \mathbf{d} + \mathbf{b}_1$, where \mathbf{b}_1 is the vector corresponding to the basic feasible solution $(12, 0)$.
- 4.] Find the null space of the matrix A and show that the vector representing the direction of unboundedness, \mathbf{d} , from the previous question is indeed in the null space of the matrix.