§4.8: Unbounded LPs

1.] Consider the following LP:

$$\text{Maximize:} \quad z = 36x_1 + 30x_2 - 3x_3 - 4x_4$$

$$\begin{aligned}\text{Subject to:} \quad x_1 + x_2 - x_3 &\leq 5 \\ 6x_1 + 5x_2 - x_4 &\leq 10 \\ x_1, x_2, x_3, x_4 &\geq 0\end{aligned}$$

a.) Solve the problem using the Simplex Method.

Row	Basic		RHS
0	z		
1			
2			

Row	Basic		RHS
0’	z		
1’			
2’			

Row	Basic		RHS
0”	z		
1”			
2”			

b.) Suppose $x_3 = 1$. How would the variables x_1 and x_4 change? How would the objective function value change? Is this solution still feasible?

c.) Let $\boldsymbol{x}$ be the the solution from the last iteration of the Simplex Method. Define the feasible solution $\boldsymbol{y} = \boldsymbol{d} + \boldsymbol{x}$ for some direction of unboundedness $\boldsymbol{d}$. Show that $\boldsymbol{c}^T\boldsymbol{d} > 0$.

2.] Consider the following LP:

$$\begin{aligned} \text{Maximize:} \quad & z = 20x_1 + 10x_2 + x_3 \\ \text{Subject to:} \quad & 3x_1 - 3x_2 + 5x_3 \le 50 \\ & x_1 \phantom{{}-3x_2} + x_3 \le 10 \\ & x_1 - x_2 + 4x_3 \le 20 \\ & x_1, x_2, x_3 \ge 0 \end{aligned}$$

(a) By inspecting the constraints, determine the direction (x_1, x_2, or x_3) in which the solution space is unbounded. Define $\boldsymbol{d}$ and show $\boldsymbol{c}^T\boldsymbol{d} > 0$.

(b) Without further computations, what can you conclude regarding the optimum objective value?

3.] Consider the following minimization LP:

$$\begin{aligned} \text{Minimize:} \quad & z = -x_1 - 3x_2 \\ \text{Subject to:} \quad & x_1 - 2x_2 \le 4 \\ & -x_1 + x_2 \le 3 \\ & x_1, x_2 \ge 0 \end{aligned}$$

Show that this LP is unbounded.

Row	Basic		RHS
0	z		
1			
2			

Row	Basic		RHS
0’	z		
1’			
2’			