

§4.13 (PART 1): THE TWO-PHASE SIMPLEX METHOD

1.] Consider the following LP:

Minimize: $z = 4x_1 + x_2$

Subject to: $3x_1 + x_2 = 3$

$4x_1 + 3x_2 \geq 6$

$x_1 + 2x_2 \leq 4$

$x_1, x_2 \geq 0$

a.) Set up Phase I and solve it.

Minimize $w = a_1 + a_2$

$$\begin{aligned} \text{Subject to: } 3x_1 + x_2 + a_1 &= 3 \\ 4x_1 + 3x_2 - e_2 + a_2 &= 6 \\ x_1 + 2x_2 + s_3 &= 4 \end{aligned}$$

$$\Rightarrow w = (3 - 3x_1 - x_2) + (6 - 4x_1 - 3x_2 + e_2) = -7x_1 - 4x_2 + e_2 + 9$$

Row	Basic	w	x_1	x_2	e_2	a_1	a_2	s_3	RHS
0	w	1	7	4	-1	0	0	0	9
1	a_1	0	3	1	0	1	0	0	3
2	a_2	0	4	3	-1	0	1	0	6
3	s_3	0	1	2	0	0	0	1	4

$$\begin{aligned} 3/3 &= 1 \leftarrow \\ 6/4 &= 3/2 \\ 4/1 &= 4 \end{aligned}$$

Row	Basic	w	x_1	x_2	e_2	a_1	a_2	s_3	RHS
0'	w	1	0	$5/3$	-1	$-7/3$	0	0	2
1'	x_1	0	1	$1/3$	0	$1/3$	0	0	1
2'	a_2	0	0	$5/3$	-1	$-4/3$	1	0	2
3'	s_3	0	0	$5/3$	0	$-1/3$	0	1	3

$$\begin{aligned} 1/3 &= 3 \leftarrow \\ 2/5 &= 2/5 \\ 3/5 &= 3/5 \end{aligned}$$

optimal!

Row	Basic	w	x_1	x_2	e_2	a_1	a_2	s_3	RHS
0''	w	1	0	0	0	-1	-1	0	0
1''	x_1	0	1	0	$1/5$	$3/5$	$-1/5$	0	$3/5$
2''	x_2	0	0	1	$-3/5$	$-4/5$	$3/5$	0	$6/5$
3''	s_3	0	0	0	1	1	-1	1	1

$$x_1 = 3/5, x_2 = 6/5, e_2 = 0$$

$$a_1 = 0, a_2 = 0, s_3 = 1$$

2.] Write down the associated LP for Phase II. Then construct the initial tableau and solve it.

Minimize $Z = 4x_1 + x_2$

Subject to: $x_1 + \frac{1}{5}e_2 = \frac{3}{5}$

$x_2 - \frac{3}{5}e_2 = \frac{6}{5}$

$e_2 + s_3 = 1$

Need to eliminate x_1 and x_2 from Row 0:

$x_1 = \frac{3}{5} - \frac{1}{5}e_2$ $x_2 = \frac{6}{5} + \frac{3}{5}e_2$

$\Rightarrow Z = \frac{12}{5} - \frac{4}{5}e_2 + \frac{6}{5} + \frac{3}{5}e_2$

$\Rightarrow Z = \frac{18}{5} - \frac{1}{5}e_2$

x_1 and x_2 are basic but have non-zero elements in Row 0!

Row	Basic	Z	x_1	x_2	e_2	s_3	RHS
0	Z	1	-4	-1	0	0	0
1	x_1	0	1	0	$\frac{1}{5}$	0	$\frac{3}{5}$
2	x_2	0	0	1	$-\frac{3}{5}$	0	$\frac{6}{5}$
3	s_3	0	0	0	1	1	1

Row	Basic	Z	x_1	x_2	e_2	s_3	RHS
0'	Z	1	0	0	$\frac{1}{5}$	0	$\frac{18}{5}$
1'	x_1	0	1	0	$\frac{1}{5}$	0	$\frac{3}{5}$
2'	x_2	0	0	1	$-\frac{3}{5}$	0	$\frac{6}{5}$
3'	s_3	0	0	0	1	1	1

$\frac{3}{1/5} = 3$

$\frac{6}{-3/5} = -2$

$\frac{1}{1} = 1$

Optimal!

Row	Basic	Z	x_1	x_2	e_2	s_3	RHS
0''	Z	1	0	0	0	$-\frac{1}{5}$	$\frac{17}{5}$
1''	x_1	0	1	0	0	$-\frac{1}{5}$	$\frac{4}{5}$
2''	x_2	0	0	1	0	$\frac{3}{5}$	$\frac{9}{5}$
3''	e_2	0	0	0	1	1	1

$x_1 = \frac{2}{5}, x_2 = \frac{9}{5}$

$e_2 = 1, s_3 = 0$

Min $Z = \frac{17}{5}$