§4.11 (PART 2): BLAND'S RULE

1.] Consider the following LP below:

$$\text{Maximize:} \quad z = -3x_1 + x_2 - 6x_3$$

$$\begin{aligned} \text{Subject to:} \quad 9x_1 + x_2 - 9x_3 - 2x_4 &\leq 0 \\ x_1 + \frac{x_2}{3} - 2x_3 - \frac{x_4}{3} &\leq 0 \\ -9x_1 - x_2 + 9x_3 + 2x_4 &\leq 1 \\ x_1, x_2, x_3, x_4 &\geq 0 \end{aligned}$$

The sequence of Simplex Tableaus (on the reverse side) show that cycling occurs if ties are broken in favor of lower numbered rows. Use Bland's Rule from the fourth iteration to show that the cycle is broken and an optimal solution is found:

Row	Basic	z	x_1	x_2	x_3	x_4	s_1	s_2	s_3	RHS
0""	z	1	0	0	3	0	−1	6	0	0
1""	x_4	0	0	−2	9	1	1	−9	0	0
2""	x_1	0	1	−⅓	1	0	⅓	−2	0	0
3""	s_3	0	0	0	0	0	1	0	1	1

$0/1 = 0$ Tie!
$0/\frac{1}{3} = 0 \leftarrow$ Bland's Choice.
$1/1 = 1$

Row	Basic	z	x_1	x_2	x_3	x_4	s_1	s_2	s_3	RHS
0""	z	1	3	−1	6	0	0	0	0	0
1""	x_4	0	−3	−1	6	1	0	−3	0	0
2""	s_1	0	3	−1	3	0	1	−6	0	0
3""	s_3	0	−3	1	−3	0	0	6	1	1

$0/-1 = ⊗$
$0/-1 = ⊗$
$1/1 = 1 \leftarrow$

Row	Basic	z	x_1	x_2	x_3	x_4	s_1	s_2	s_3	RHS
0""	z	1	6	0	3	0	0	6	1	1
1""	x_4	0	−6	0	3	1	0	3	1	1
2""	s_1	0	0	0	0	0	1	0	1	1
3""	x_2	0	−3	1	−3	0	0	6	1	1

optimal!

Solution: $x_1 = 0,\ x_2 = 1,\ x_3 = 0,\ x_4 = 1$

$s_1 = 1,\ s_2 = 0,\ s_3 = 0$

Max $z = 1$

Row	Basic	z	x_1	x_2	x_3	x_4	s_1	s_2	s_3	RHS	
0	z	1	3	-1	6	0	0	0	0	0	
1	s_1	0	9	1	-9	-2	1	0	0	0	$0/1 = 0$ ←
2	s_2	0	1	1/3	-2	-1/3	0	1	0	0	$0/\frac{1}{3} = 0$
3	s_3	0	-9	-1	9	2	0	0	1	1	$1/1 = \otimes$

Tie

Row	Basic	z	x_1	x_2	x_3	x_4	s_1	s_2	s_3	RHS	
0'	z	1	12	0	-3	-2	1	0	0	0	
1'	x_2	0	9	1	-9	-2	1	0	0	0	$0/9 = 0$
2'	s_2	0	-2	0	1	1/3	-1/3	1	0	0	$0/1 = 0$ ←
3'	s_3	0	0	0	0	0	1	0	1	1	$1/0 = \otimes$

Row	Basic	z	x_1	x_2	x_3	x_4	s_1	s_2	s_3	RHS	
0"	z	1	6	0	0	-1	0	3	0	0	
1"	x_2	0	-9	1	0	1	-2	9	0	0	$0/1 = 0$ ←
2"	x_3	0	-2	0	1	1/3	-1/3	1	0	0	$0/\frac{1}{3} = 0$
3"	s_3	0	0	0	0	0	1	0	1	1	$1/0 = \otimes$

Tie!

Row	Basic	z	x_1	x_2	x_3	x_4	s_1	s_2	s_3	RHS	
0"'	z	1	-3	1	0	0	-2	12	0	0	
1"'	x_4	0	-9	1	0	1	-2	9	0	0	$0/9 = \otimes$
2"'	x_3	0	1	-1/3	1	0	1/3	-2	0	0	$0/1 = 0$ ←
3"'	s_3	0	0	0	0	0	1	6	1	1	$1/0 = \otimes$

Row	Basic	z	x_1	x_2	x_3	x_4	s_1	s_2	s_3	RHS	
0""	z	1	0	0	3	0	-1	6	0	0	
1""	x_4	0	0	-2	9	1	1	-9	0	0	$0/1 = 0$ ←
2""	x_1	0	1	-1/3	1	0	1/3	-2	0	0	$0/\frac{1}{3} = 0$
3""	s_3	0	0	0	0	0	1	0	1	1	$1/1 = 1$

Tie!

Row	Basic	z	x_1	x_2	x_3	x_4	s_1	s_2	s_3	RHS	
0""	z	1	0	-2	12	1	0	-3	0	0	
1""	s_1	0	0	-2	9	1	1	-9	0	0	$0/9 = \otimes$
2""	x_1	0	1	1/3	-2	-1/3	0	1	0	0	$0/1 = 0$ ←
3""	s_3	0	0	2	-9	-1	0	9	1	1	$1/9$

Row	Basic	z	x_1	x_2	x_3	x_4	s_1	s_2	s_3	RHS
0""	z	1	3	-1	6	0	0	0	0	0
1""	s_1	0	9	1	-9	-2	1	0	0	0
2""	s_2	0	1	1/3	-2	-1/3	0	1	0	0
3""	s_3	0	-9	-1	9	2	0	0	1	1