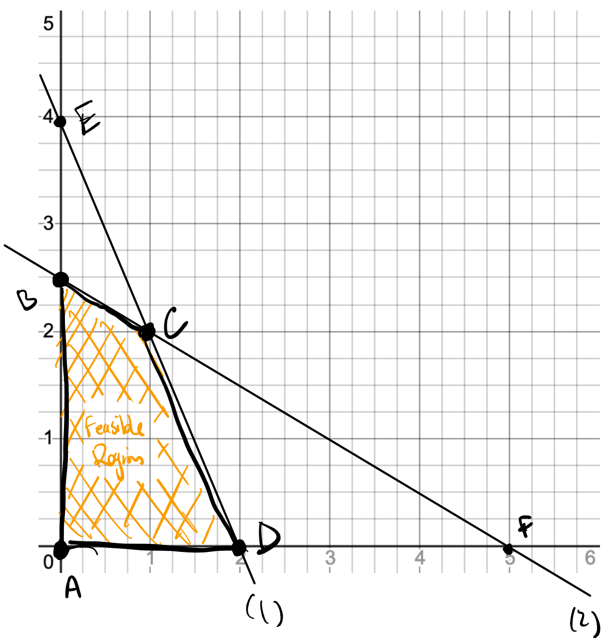
§4.2: Preview of the Simplex Algorithm

1.] Consider the following two variable LP below:

$$\text{Maximize:} \quad z = 2x_1 + 3x_2$$

$$\begin{aligned} \text{Subject to:} \quad 2x_1 + x_2 &\leq 4 \\ x_1 + 2x_2 &\leq 5 \\ x_1, x_2 &\geq 0 \end{aligned}$$

Sketch the feasible space on the graph below, labeling the corner points of the feasible space. Convert the LP to standard form and identify all basic solutions with their corresponding objective values by filling out the table below.



Constraint Lines:

(1) $x_2 = -2x_1 + 4 \longrightarrow (0,4)$ & $(2,0)$

(2) $x_2 = -\frac{1}{2}x_1 + \frac{5}{2} \longrightarrow (0, 5/2)$ & $(5,0)$

Standard Form: max $z = 2x_1 + 3x_2$

Subject to:

$$\boxed{\begin{aligned} 2x_1 + x_2 + s_1 \quad &= 4 \\ x_1 + 2x_2 \quad + s_2 &= 5 \end{aligned}}$$

Use to find Basic Solution

$x_1, x_2, s_1, s_2 \geq 0$

$$A = \begin{bmatrix} 2 & 1 & 1 & 0 \\ 1 & 2 & 0 & 1 \end{bmatrix}$$

$n = 4$
$m = 2$

# of NonBasic Vars: $n - m = 2$
# of Basic Vars: $m = 2$

Nonbasic Variables	Basic Variables	Basic Solution	Corner Point	Feasible?	Obj Value (z)
x_1, x_2	s_1, s_2	$s_1 = 4, s_2 = 5$	A	Yes	$z = 0$
x_1, s_1	x_2, s_2	$x_2 = 4, s_2 = -3$	E	No	—
x_1, s_2	x_2, s_1	$x_2 = 5/2, s_1 = 3/2$	B	Yes	$z = 15/2$
x_2, s_1	x_1, s_2	$x_1 = 2, s_2 = 3$	D	Yes	$z = 4$
x_2, s_2	x_1, s_1	$x_1 = 5, s_1 = -6$	F	No	—
s_1, s_2	x_1, x_2	$x_1 = 1, x_2 = 2$	C	Yes	$z = 8$

OPTIMAL

2.] Consider the following two variable LP below:

$$\text{Maximize:} \quad z = 2x_1 - 4x_2 + 5x_3 - 6x_4$$

$$\begin{aligned} \text{Subject to:} \quad x_1 + 4x_2 - 2x_3 + 8x_4 &\leq 2 \\ -x_1 + 2x_2 + 3x_3 + 4x_4 &\leq 1 \\ x_1, x_2, x_3, x_4 &\geq 0 \end{aligned}$$

Convert the LP to standard form and identify all basic solutions with their corresponding objective values by explicitly listing all sets of nonbasic and basic variables and determining the basic solution for each set. For each basic solution, identify the objective function value and whether or not it is feasible. From your listing, determine the optimal solution.

Standard Form:

Maximize $z = 2x_1 - 4x_2 + 5x_3 - 6x_4$

Subject to:

$$\begin{aligned} x_1 + 4x_2 - 2x_3 + 8x_4 + s_1 \quad &= 2 \\ -x_1 + 2x_2 + 3x_3 + 4x_4 \quad + s_2 &= 1 \\ x_1, x_2, x_3, x_4, x_5, x_6, s_1, s_2 &\geq 0 \end{aligned}$$

$$A = \begin{bmatrix} 1 & 4 & -2 & 8 & 1 & 0 \\ -1 & 2 & 3 & 4 & 0 & 1 \end{bmatrix}, \quad \vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ s_1 \\ s_2 \end{bmatrix}, \text{ and } \vec{b} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

# of Equs/Constraints: $m = 2$

# of Variables: $n = 6$

NBVs: $n - m = 4$

BVs: $m = 2$

of Basic Sol's:

$_nC_m = {}_6C_2 = \frac{6!}{2!4!} = 15$

	NBV	BV	Basic Solution (See Below)	Feasible?	Obj Fun
1	x_1, x_2, x_3, x_4	s_1, s_2	$s_1 = 2, s_2 = 1$	Yes	$z = 0$
2	x_1, x_2, x_3, s_1	x_4, s_2	$x_4 = 1/4, s_2 = 0$	Yes	$z = -3/2$
3	x_1, x_2, x_3, s_2	x_4, s_1	$x_4 = 1/4, s_1 = 0$	Yes	$z = -3/2$
4	x_1, x_2, s_1, x_4	x_3, s_2	$x_3 = -1, s_2 = 4$	No	—
5	x_1, x_2, s_2, x_4	x_3, s_1	$x_3 = 1/3, s_1 = 8/3$	Yes	$z = 5/3$
6	x_1, s_1, x_3, x_4	x_2, s_2	$x_2 = 1/2, s_2 = 0$	Yes	$z = -2$
7	x_1, s_2, x_3, x_4	x_2, s_1	$x_2 = 1/2, s_1 = 0$	Yes	$z = -2$
8	s_1, x_2, x_3, x_4	x_1, s_2	$x_1 = 2, s_2 = 3$	Yes	$z = 4$
9	s_2, x_2, x_3, x_4	x_1, s_1	$x_1 = -1, s_1 = 3$	No	—
10	x_1, x_2, s_1, s_2	x_3, x_4	$x_3 = 0, x_4 = 1/4$	Yes	$z = -3/2$
11	x_1, s_1, x_3, s_2	x_2, x_4	$x_2 = 1/2, x_4 = 0$ (not unique)	Yes	$z = -2$
12	s_1, x_2, x_3, s_2	x_1, x_4	$x_1 = 0, x_4 = 1/4$	Yes	$z = -3/2$
13	x_1, s_1, s_2, x_4	x_2, x_3	$x_2 = 1/2, x_3 = 0$	Yes	$z = -2$
14	s_1, x_2, s_2, x_4	x_1, x_3	$x_1 = 8, x_3 = 3$	Yes	$z = 31$ ← OPTIMAL
15	s_1, s_2, x_3, x_4	x_1, x_2	$x_1 = 0, x_2 = 1/2$	Yes	$z = -2$

1.) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} s_1 \\ s_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \longrightarrow s_1 = 2, s_2 = 1$

2.) $\begin{bmatrix} 8 & 0 \\ 4 & 1 \end{bmatrix}\begin{bmatrix} x_4 \\ s_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \longrightarrow x_4 = 1/4, s_2 = 0$

3.) $\begin{bmatrix} 8 & 1 \\ 4 & 0 \end{bmatrix}\begin{bmatrix} x_4 \\ s_1 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \longrightarrow x_4 = 1/4, s_1 = 0$

4.) $\begin{bmatrix} -2 & 0 \\ 3 & 1 \end{bmatrix}\begin{bmatrix} x_3 \\ s_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \longrightarrow x_3 = -1, s_2 = 4$

5.) $\begin{bmatrix} -2 & 1 \\ 3 & 0 \end{bmatrix}\begin{bmatrix} x_3 \\ s_1 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \longrightarrow x_3 = 1/3, s_1 = 8/3$

6.) $\begin{bmatrix} 4 & 0 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} x_2 \\ s_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \longrightarrow x_2 = 1/2, s_2 = 0$

7.) $\begin{bmatrix} 4 & 1 \\ 2 & 0 \end{bmatrix}\begin{bmatrix} x_2 \\ s_1 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \longrightarrow x_2 = 1/2, s_1 = 0$

8.) $\begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ s_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \longrightarrow x_1 = 2, s_2 = 3$

9.) $\begin{bmatrix} 1 & 1 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ s_1 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \longrightarrow x_1 = -1, s_1 = 3$

10.) $\begin{bmatrix} -2 & 8 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \longrightarrow x_3 = 0, x_4 = 1/4$

11.) $\begin{bmatrix} 4 & 8 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} x_2 \\ x_4 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \longrightarrow x_2 = 1/2, x_4 = 0$
(not unique)

12.) $\begin{bmatrix} 1 & 8 \\ -1 & 4 \end{bmatrix}\begin{bmatrix} x_1 \\ x_4 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \longrightarrow x_1 = 0, x_4 = 1/4$

13.) $\begin{bmatrix} 4 & -2 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \longrightarrow x_2 = 1/2, x_3 = 0$

14.) $\begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \longrightarrow x_1 = 8, x_3 = 3$

15.) $\begin{bmatrix} 1 & 4 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \longrightarrow x_1 = 0, x_2 = 1/2$