§4.6: Simplex Algorithm for Minimization LPs

1.] Solve the following two-variable LP problem using the Simplex Method:

Minimize: $z = 4x_1 - x_2$ $\Rightarrow$ $-z = -4x_1 + x_2$

$\Rightarrow$ $-z + 4x_1 - x_2 = 0$

Subject to:
$$2x_1 + x_2 \le 8$$
$$x_2 \le 5$$
$$x_1 - x_2 \le 4$$
$$x_1, x_2 \ge 0$$

Row	Basic	$-z$	x_1	x_2	s_1	s_2	s_3	RHS	
0	$-z$	1	4	-1	0	0	0	0	
1	s_1	0	2	1	1	0	0	8	$8/1 = 8$
2	s_2	0	0	1	0	1	0	5	$5/1 = 5$ ←
3	s_3	0	1	-1	0	0	1	4	$4/-1 =$ ⊗

Optimal!

Row	Basic	$-z$	x_1	x_2	s_1	s_2	s_3	RHS
0'	$-z$	1	4	0	0	1	0	5
1'	s_1	0	2	0	1	-1	0	3
2'	x_2	0	0	1	0	1	0	5
3'	s_3	0	1	0	0	1	1	9

$x_1 = 0, x_2 = 5$

$s_1 = 3, s_2 = 0, s_3 = 9$

Max $-z = 5$

$\Rightarrow$ Min $z = -5$

2.] Solve the following two-variable LP problem using the Simplex Method:

Minimize: $z = 2x_1 - 5x_2$ → $z - 2x_1 + 5x_2 = 0$

Subject to: $3x_1 + 8x_2 \leq 12$
$2x_1 + 3x_2 \leq 6$
$x_1, x_2 \geq 0$

Row	Basic	z	x_1	x_2	s_1	s_2	RHS
0	z	1	−2	5	0	0	0
1	s_1	0	3	8	1	0	12
2	s_2	0	2	3	0	1	6

$12/8 = 3/2$ ←
$6/3 = 2$

Optimal!

Row	Basic	z	x_1	x_2	s_1	s_2	RHS
0'	z	1	$-31/8$	0	$-5/8$	0	$-15/2$
1'	x_2	0	$3/8$	1	$1/8$	0	$3/2$
2'	s_2	0	$7/8$	0	$-3/8$	1	$3/2$

$x_1 = 0, \quad x_2 = 3/2$

$s_1 = 0, \quad s_2 = 3/2$

Min $z = -15/2$