§4.5 (part 1): Simplex Algorithm

1.] Reddy Mikks Company: Recall the Reddy Mikks company once again. Below the LP is in standard form. Follow the steps below to solve this problem using the Simplex Algorithm.

$$\text{Maximize:} \quad z = 5x_1 + 4x_2$$

$$\begin{aligned} \text{Subject to:} \quad 6x_1 + 4x_2 + s_1 &= 24 \\ x_1 + 2x_2 + s_2 &= 6 \\ -x_1 + x_2 + s_3 &= 1 \\ x_2 + s_4 &= 2 \\ x_1, x_2, s_1, s_2, s_3, s_4 &\geq 0 \end{aligned}$$

a.) Letting x_1 and x_2 be the nonbasic variables, set up the initial Simplex Tableau in the table below.

Row	Basic	z	x_1	x_2	s_1	s_2	s_3	s_4	RHS
0	z	1	-5	-4	0	0	0	0	0
1	s_1	0	$\boxed{6}$	4	1	0	0	0	24
2	s_2	0	1	2	0	1	0	0	6
3	s_3	0	-1	1	0	0	1	0	1
4	s_4	0	0	1	0	0	0	1	2

b.) Identify the *entering variable* and highlight the *pivot column*. Why is this the entering variable?

We choose x_1 to be the entering variable since it corresponds to the most negative coefficient in Row 0 i.e. increasing x_1 by 1 unit increases the obj. fun. value by 5 units. Pivot Col: x_1

c.) Determine the *pivot row* and the *pivot element* by identifying the *leaving variable*, which is determined by the *ratio test*:

Pivot Row: Row 1
Pivot Element: 6
Leaving Var: s_1

Basic	Entering x_1	RHS	Ratio
s_1	6	24	$\longrightarrow$ 24/6 = 4 ← Minimum, winner!
s_2	1	6	$\longrightarrow$ 6/1 = 6
s_3	-1	1	$\longrightarrow$ 1/-1 = -1 ← Ignore neg. values
s_4	0	2	$\longrightarrow$ 2/0 = ∞ ← Definitely ignore

d.) Construct the new pivot row by renaming the leaving variable as the entering variable and dividing the entire row by the pivot element.

$$\text{Row } 1' \; \| x_1 \| \; \frac{1}{6}\begin{bmatrix} 0 & 6 & 4 & 1 & 0 & 0 & 0 & 24 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 2/3 & 1/6 & 0 & 0 & 0 & 4 \end{bmatrix}$$

e.) Construct the new rows:

- Construct Row 0':

$$\text{Row } 0' \,|\, z \,|\, \overbrace{[1\ -5\ -4\ 0\ 0\ 0\ 0\ 0]}^{\text{OLD Row 0}} - \underset{\substack{\text{Pivot Col Element} \\ \text{in Row 0}}}{(-5)} \overbrace{[0\ 1\ 2/3\ 1/6\ 0\ 0\ 0\ 4]}^{\text{New Pivot Row}}$$

$$= [1\ 0\ -2/3\ 5/6\ 0\ 0\ 0\ 20]$$

- Construct Row 2':

$$\text{Row } 2' \,|\, s_2 \,|\, [0\ 1\ 2\ 0\ 1\ 0\ 0\ 6] - (1)[0\ 1\ 2/3\ 1/6\ 0\ 0\ 0\ 4]$$

$$= [0\ 0\ 4/3\ -1/6\ 1\ 0\ 0\ 2]$$

- Construct Row 3':

$$\text{Row } 3' \,|\, s_3 \,|\, [0\ -1\ 1\ 0\ 0\ 1\ 0\ 1] - (-1)[0\ 1\ 2/3\ 1/6\ 0\ 0\ 0\ 4]$$

$$= [0\ 0\ 5/3\ 1/6\ 0\ 1\ 0\ 5]$$

- Construct Row 4':

$$\text{Row } ' \,|\, s \,|\, [0\ 0\ 1\ 0\ 0\ 0\ 1\ 2] - (0)[0\ 1\ 2/3\ 1/6\ 0\ 0\ 0\ 4]$$

$$= [0\ 0\ 1\ 0\ 0\ 0\ 1\ 2]$$

f.) Assemble the Simplex Tableau. The first iteration is complete.

Row	Basic	z	x_1	x_2	s_1	s_2	s_3	s_4	RHS
0'	z	1	0	$-2/3$	$5/6$	0	0	0	20
1'	x_1	0	1	$2/3$	$1/6$	0	0	0	4
2'	s_2	0	0	$\boxed{4/3}$	$-1/6$	1	0	0	2
3'	s_3	0	0	$5/3$	$1/6$	0	1	0	5
4'	s_4	0	0	1	0	0	0	1	2

g.) Examine the tableau and determine the current value of the objective function. Is it optimal? If not, identify the next entering variable.

With $x_1=4$ and $x_2=0$, the current value of the objective function is $z=20$. This is not optimal because if $x_2=1$, the obj fun will increase by $2/3$ units. Therefore, x_2 is the next entering variable.

h.) Determine the leaving variable, pivot row, and pivot element by performing the ratio test:

Basic	Entering x_2	RHS	Ratio
x_1	$2/3$	4	$\longrightarrow$ $4/\frac{2}{3} = 6$
s_2	$4/3$	2	$\longrightarrow$ $2/\frac{4}{3} = 3/2$ ← winner, s_2 is the leaving var.
s_3	$5/3$	5	$\longrightarrow$ $5/\frac{5}{3} = 3$
s_4	1	2	$\longrightarrow$ $2/1 = 2$

i.) Construct the new pivot row by renaming the leaving variable as the entering variable and dividing the entire row by the pivot element.

$$\text{Row } 2'' \,|\, x_2 \,|\, \frac{3}{4}\begin{bmatrix} 0 & 0 & \frac{4}{3} & -\frac{1}{6} & 1 & 0 & 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 1 & -\frac{1}{8} & \frac{3}{4} & 0 & 0 & \frac{3}{2} \end{bmatrix}$$

j.) Construct the new rows:

- Construct Row 0”:

$$\text{Row } 0'' \,|\, z \,|\, \begin{bmatrix} 1 & 0 & -2/3 & 5/6 & 0 & 0 & 0 & 20 \end{bmatrix} - (-2/3)\begin{bmatrix} 0 & 0 & 1 & -\frac{1}{8} & \frac{3}{4} & 0 & 0 & \frac{3}{2} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 3/4 & 1/2 & 0 & 0 & 21 \end{bmatrix}$$

- Construct Row 1”:

$$\text{Row } 1'' \,|\, x_1 \,|\, \begin{bmatrix} 0 & 1 & 2/3 & 1/6 & 0 & 0 & 0 & 4 \end{bmatrix} - (2/3)\begin{bmatrix} 0 & 0 & 1 & -\frac{1}{8} & \frac{3}{4} & 0 & 0 & \frac{3}{2} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 0 & 1/4 & -1/2 & 0 & 0 & 3 \end{bmatrix}$$

- Construct Row 3”:

$$\text{Row } 3'' \,|\, s_3 \,|\, \begin{bmatrix} 0 & 0 & 5/3 & 1/6 & 0 & 1 & 0 & 5 \end{bmatrix} - (5/3)\begin{bmatrix} 0 & 0 & 1 & -\frac{1}{8} & \frac{3}{4} & 0 & 0 & \frac{3}{2} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 & 9/24 & -5/4 & 1 & 0 & 5/2 \end{bmatrix}$$

- Construct Row 4”:

$$\text{Row } 4'' \,|\, s_4 \,|\, \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 1 & 2 \end{bmatrix} - (1)\begin{bmatrix} 0 & 0 & 1 & -\frac{1}{8} & \frac{3}{4} & 0 & 0 & \frac{3}{2} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 & 1/8 & -3/4 & 0 & 1 & 1/2 \end{bmatrix}$$

k.) Assemble the Simplex Tableau. The second iteration is complete.

Row	Basic	z	x_1	x_2	s_1	s_2	s_3	s_4	RHS
0”	z	1	0	0	3/4	1/2	0	0	21
1”	x_1	0	1	0	1/4	-1/2	0	0	3
2”	x_2	0	0	1	-1/8	3/4	0	0	3/2
3”	s_3	0	0	0	9/24	-5/4	1	0	5/2
4”	s_4	0	0	0	1/8	-3/4	0	1	1/2

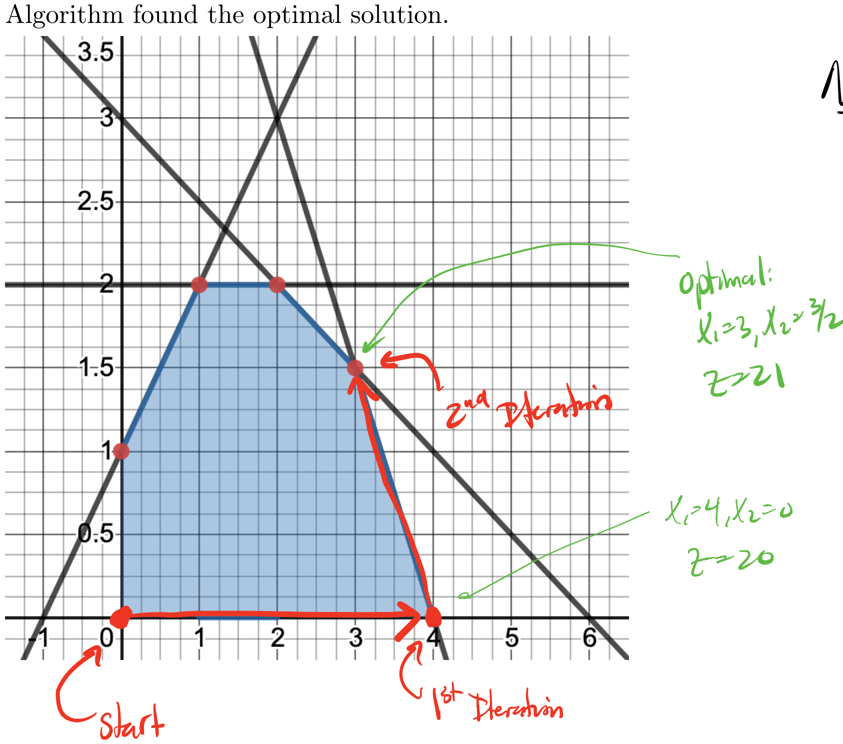
l.) Examine the tableau and determine the current value of the objective function. Report the optimal solution and the maximum objective function value.

This tableau is optimal since introducing any new variables will decrease the obj. fun. value. The optimal solution is

$x_1 = 3, \ x_2 = 3/2$

and the optimal obj. fun. value is $z = 21$.

m.) Below is the feasible space of the Reddy Mikks problem. Identify the path for which the Simplex Algorithm found the optimal solution.



Note: the ratios of the ratio test point to intercepts of the constraints.