

§4.5 (PART 2): SIMPLEX ALGORITHM

1.] Solve the following two-variable LP problem using the Simplex Method:

$$\text{Maximize: } z = 2x_1 + 3x_2$$

$$\text{Subject to: } x_1 + 2x_2 \leq 6$$

$$2x_1 + x_2 \leq 8$$

$$x_1, x_2 \geq 0$$

Row	Basic	z	x_1	x_2	s_1	s_2	RHS
0	z	1	-2	-3	0	0	0
1	s_1	0	1	2	1	0	6
2	s_2	0	2	1	0	1	8

Ratios

$$6/2 = 3 \leftarrow \text{leaving}$$

$$8/1 = 8$$

Row	Basic	z	x_1	x_2	s_1	s_2	RHS
0'	z	1	-1/2	0	3/2	0	9
1'	x_2	0	1/2	1	1/2	0	3
2'	s_2	0	3/2	0	-1/2	1	5

Ratios

$$3/(1/2) = 6$$

$$5/(3/2) = 10/3 \leftarrow \text{leaving}$$

Row	Basic	z	x_1	x_2	s_1	s_2	RHS
0''	z	1	0	0	4/3	1/3	32/3
1''	x_2	0	0	1	2/3	-1/3	4/3
2''	x_1	0	1	0	-1/3	2/3	10/3

Optimal

Optimal values: $x_1 = \frac{10}{3}$, $x_2 = \frac{4}{3}$, $s_1 = 0$, $s_2 = 0$

$$\text{Max Obj fun. value: } z = \frac{32}{3}$$

2.] Solve the following two-variable LP problem using the Simplex Method:

$$\text{Maximize: } z = 2x_1 - x_2 + x_3$$

$$\text{Subject to: } 3x_1 + x_2 + x_3 \leq 60$$

$$x_1 - x_2 + 2x_3 \leq 10$$

$$x_1 + x_2 - x_3 \leq 20$$

$$x_1, x_2, x_3 \geq 0$$

Row	Basic	z	x_1	x_2	x_3	s_1	s_2	s_3	RHS
0	z	1	-2	1	-1	0	0	0	0
1	s_1	0	3	1	1	1	0	0	60
2	s_2	0	1	-1	2	0	1	0	10
3	s_3	0	1	1	-1	0	0	1	20

$60/3 = 20$
 $10/1 = 10 \leftarrow \text{leaving}$
 $20/1 = 20$

not optimal

Row	Basic	z	x_1	x_2	x_3	s_1	s_2	s_3	RHS
0'	z	1	0	-1	3	0	2	0	20
1'	s_1	0	0	4	-5	1	-3	0	30
2'	x_1	0	1	-1	2	0	1	0	10
3'	s_3	0	0	2	-3	0	-1	1	10

$30/4 = 7.5$
 $10/1 \leftarrow \text{Ignore}$
 $10/2 = 5 \leftarrow \text{leaving}$

Optimal!

Row	Basic	z	x_1	x_2	x_3	s_1	s_2	s_3	RHS
0''	z	1	0	0	$3/2$	0	$3/2$	$1/2$	25
1''	s_1	0	0	0	1	1	-1	-2	10
2''	x_1	0	1	0	$-1/2$	0	$1/2$	$1/2$	15
3''	x_2	0	0	1	$-3/2$	0	$-1/2$	$1/2$	5

$x_1 = 15, x_2 = 5, x_3 = 0$
 $s_1 = 10, s_2 = 0, s_3 = 0$
 $\max z = 25$

Row	Basic	z	x_1	x_2	x_3	s_1	s_2	s_3	RHS
0'''	z								
1'''									
2'''									
3'''									

UNUSED

Row	Basic	z	x_1	x_2	x_3	s_1	s_2	s_3	RHS
0''''	z								
1''''									
2''''									
3''''									

UNUSED