

## §4.1: STANDARD FORM

- 1.] REDDY MIKKS COMPANY: Recall the Reddy Mikks company (Section 3.2) that produces both interior and exterior paints from two raw materials. Below is the original LP formulation:

$$\text{Maximize: } z = 5x_1 + 4x_2$$

$$\text{Subject to: } 6x_1 + 4x_2 \leq 24$$

$$x_1 + 2x_2 \leq 6$$

$$-x_1 + x_2 \leq 1$$

$$x_2 \leq 2$$

$$x_1, x_2 \geq 0$$

Convert the problem to standard form and determine the number of equations,  $m$ , and the number of variables  $n$ . Determine the matrix  $A$  and the vectors  $\mathbf{x}$  and  $\mathbf{b}$ .

Standard Form: Maximize  $z = 5x_1 + 4x_2$

Subject to:

$$\begin{aligned} 6x_1 + 4x_2 + s_1 &= 24 \\ x_1 + 2x_2 + s_2 &= 6 \\ -x_1 + x_2 + s_3 &= 1 \\ x_2 + s_4 &= 2 \\ x_1, x_2, s_1, s_2, s_3, s_4 &\geq 0 \end{aligned}$$

Matrix/Vector Form: The constraints take the form  $A\vec{x} = \vec{b}$ , with

$$A = \begin{bmatrix} 6 & 4 & 1 & 0 & 0 & 0 \\ 1 & 2 & 0 & 1 & 0 & 0 \\ -1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad \vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ s_1 \\ s_2 \\ s_3 \\ s_4 \end{bmatrix}, \quad \text{and} \quad \vec{b} = \begin{bmatrix} 24 \\ 6 \\ 1 \\ 2 \end{bmatrix}$$

- # of Constraints/Equations:  $m = 4$
- # of Decision Variables:  $n = 6$
- $A$  is  $m \times n$ ,  $\vec{x}$  is  $n \times 1$ , and  $\vec{b}$  is  $m \times 1$ .

2.] HEART VALVES: Recall U.S. Labs company that manufactures mechanical heart valves from the heart valves of pigs (Section 3.4). Below is the original LP formulation:

$$\text{Maximize: } z = 5x_1 + 4x_2 + 3x_3$$

$$\text{Subject to: } .40x_1 + .30x_2 + .20x_3 \geq 500$$

$$.40x_1 + .35x_2 + .20x_3 \geq 300$$

$$.20x_1 + .35x_2 + .60x_3 \geq 300$$

$$x_1 \leq 700$$

$$x_2 \leq 700$$

$$x_3 \leq 700$$

$$x_1, x_2, x_3 \geq 0$$

Convert the problem to standard form and determine the number of equations,  $m$ , and the number of variables  $n$ . Determine the matrix  $A$  and the vectors  $x$  and  $b$ .

Standard Form: Maximize  $Z = 5x_1 + 4x_2 + 3x_3$

Subject to:

$$.40x_1 + .30x_2 + .20x_3 - e_1 = 500$$

$$.40x_1 + .35x_2 + .20x_3 - e_2 = 300$$

$$.20x_1 + .35x_2 + .60x_3 - e_3 = 300$$

$$x_1 + s_1 = 700$$

$$x_2 + s_2 = 700$$

$$x_3 + s_3 = 700$$

$$x_1, x_2, x_3, e_1, e_2, e_3, s_1, s_2, s_3 \geq 0$$

Matrix/Vector Form: The constraints take the form  $A\vec{x} = \vec{b}$ , with

$$A = \begin{bmatrix} .40 & .30 & .20 & -1 & 0 & 0 & 0 & 0 & 0 \\ .40 & .35 & .20 & 0 & -1 & 0 & 0 & 0 & 0 \\ .20 & .35 & .60 & 0 & 0 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad \vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ e_1 \\ e_2 \\ e_3 \\ s_1 \\ s_2 \\ s_3 \end{bmatrix}, \quad \text{and } \vec{b} = \begin{bmatrix} 500 \\ 300 \\ 300 \\ 700 \\ 700 \\ 700 \end{bmatrix}$$

• # of Constraints/Equations:  $m = 6$

• # of Decision Variables:  $n = 9$

•  $A$  is  $m \times n$ ,  $\vec{x}$  is  $n \times 1$ , and  $\vec{b}$  is  $m \times 1$ .