# §3.12: Multi-Period Scheduling

1.] Computer Service: CSL is a chain of computer service stores. The number of hours of skilled repair time that CSL requires during the next five months is given in the table below. At the beginning of January, 50 skilled technicians work for CSL. Each skilled technician can work up to 160 hours per month. To meet future demands, new technicians must be trained. It takes one month to train a new technician. During the month of training, a trainee must be supervised for 50 hours by an experienced technician. Each experienced technician is paid $2,000 a month (even if he or she does not work the full 160 hours). During the month of training, a trainee is paid $1,000 a month. At the end of each month 5% of CSL's experienced technicians quit to join Plum Computers. Formulate an LP whose solution will enable CSL to minimize the labor cost incurred in meeting the service requirements for the next five months.

| Month | Required Hours |
|---|---|
| January | 6,000 |
| February | 7,000 |
| March | 8,000 |
| April | 9,500 |
| May | 11,000 |

Decision Variables: $x_t$ = trainees trained during month $t$

$y_t$ = # of experienced techs at beginning of month $t$.

Obj. Function: Minimize Labor Cost

$$Z = 1000(x_1+x_2+x_3+x_4+x_5) + 2000(y_1+y_2+y_3+y_4+y_5)$$

Constraints:

(Tech Hour Requirements)

$$160y_1 - 50x_1 \geq 6000$$
$$160y_2 - 50x_2 \geq 7000$$
$$160y_3 - 50x_3 \geq 8000$$
$$160y_4 - 50x_4 \geq 9500$$
$$160y_5 - 50x_5 \geq 11000$$

(Inventory of Skilled techs from month to month)

$y_1 = 50$

$y_2 = y_1 + x_1 - 0.05y_1$ ← percentage leaving

$y_3 = y_2 + x_2 - 0.05y_2$

$y_4 = y_3 + x_3 - 0.05y_3$

$y_5 = y_4 + x_4 - 0.05y_4$

Solution: From Excel:

| | |
|---|---|
| $x_1 = 0$ | $y_1 = 50$ |
| $x_2 = 8.453$ | $y_2 = 47.5$ |
| $x_3 = 11.45$ | $y_3 = 53.58$ |
| $x_4 = 9.518$ | $y_4 = 62.35$ |
| $x_5 = 0$ | $y_5 = 68.75$ |

$\min Z = 593,776.51$

2.] Wheat Warehouse: You own a wheat warehouse with a capacity of 20,000 bushels. At the beginning of month 1, you have 6,000 bushels of wheat. Each month, wheat can be bought and sold at the price per 1000 bushels given in the table below. The sequence of events during each month is as follows:

- You observe your initial stock of wheat.
- You can sell any amount of wheat up to your initial stock at the current month's selling price.
- You can buy (at the current month's buying price) as much wheat as you want, subject to the warehouse size limitation.

Your goal is to formulate an LP that can be used to determine how to maximize the profit earned over the next 10 months.

| Month | Selling Price ($) | Purchase Price ($) |
|---|---|---|
| 1 | 3 | 8 |
| 2 | 6 | 8 |
| 3 | 7 | 2 |
| 4 | 1 | 3 |
| 5 | 4 | 4 |
| 6 | 5 | 3 |
| 7 | 5 | 3 |
| 8 | 1 | 2 |
| 9 | 3 | 5 |
| 10 | 2 | 5 |

Decision Variables:

$i_t$ = inventory of wheat at the end of month $t$.

$x_t$ = wheat bought during month $t$.

$s_t$ = wheat sold during month $t$

Obj Fun: Max Profit:

$$Z = (3s_1 + 6s_2 + 7s_3 + s_4 + 4s_5 + 5s_6 + 5s_7 + s_8 + 3s_9 + 2s_{10}) - \cdots$$
$$(8x_1 + 8x_2 + 2x_3 + 3x_4 + 4x_5 + 3x_6 + 3x_7 + 2x_8 + 5x_9 + 5x_{10})$$

Constraints:

(Max Selling Capacity)

$s_1 \le i_0$
$s_2 \le i_1$
$s_3 \le i_2$
$s_4 \le i_3$
$s_5 \le i_4$
$s_6 \le i_5$
$s_7 \le i_6$
$s_8 \le i_7$
$s_9 \le i_8$
$s_{10} \le i_9$

(Inventory Limits)

$i_0 = 6000$
$i_1 \le 20000$
$i_2 \le 20000$
$i_3 \le 20000$
$i_4 \le 20000$
$i_5 \le 20000$
$i_6 \le 20000$
$i_7 \le 20000$
$i_8 \le 20000$
$i_9 \le 20000$
$i_{10} \le 20000$

(Inventory)

$i_1 = i_0 + x_1 - s_1$
$i_2 = i_1 + x_2 - s_2$
$i_3 = i_2 + x_3 - s_3$
$i_4 = i_3 + x_4 - s_4$
$i_5 = i_4 + x_5 - s_5$
$i_6 = i_5 + x_6 - s_6$
$i_7 = i_6 + x_7 - s_7$
$i_8 = i_7 + x_8 - s_8$
$i_9 = i_8 + x_9 - s_9$
$i_{10} = i_9 + x_{10} - s_{10}$

Solution: From Excel:

$x_1 = x_2 = x_4 = x_5 = x_7 = x_9 = x_{10} = 0$

$x_3 = x_6 = x_8 = 20{,}000$

$s_1 = s_2 = s_4 = s_5 = s_8 = s_{10} = 0$

$s_3 = 6{,}000$, $s_6 = s_7 = s_9 = 20{,}000$

$i_1 = i_2 = 6000$, $i_3 = i_4 = i_5 = i_6 = i_8 = 20{,}000$ $i_7 = i_9 = i_{10} = 0$

Max $Z = \$162{,}000$