§3.9: Production Process Problems

1.] Perfume Production: Rylon Corporation manufactures Brute and Chanelle perfumes. The raw material needed to manufacture each type of perfume can be purchased for \$3/lb. Processing 1 lb of raw material requires 1 hr of laboratory time. Each pound of processed raw material yields 3 oz of Regular Brute Perfume and 4 oz of Regular Chanelle Perfume. Regular Brute can be sold for \$7/oz and Regular Chanelle for \$6/oz. Rylon also has the option of further processing Regular Brute and Regular Chanelle to produce Luxury Brute, sold at \$18/oz and Luxury Chanelle, sold at \$14/oz. Each ounce of Regular Brute processed further requires an additional 3 hrs of laboratory time and \$4 processing cost and yields 1 oz of Luxury Brute. Each ouce of Regular Chanelle processed further requires an additional 2 hrs of laboratory time and \$4 processing cost and yields 1 oz of Luxury Chanelle. Rylon has 6,000 hrs of laboratory time available and can purchase up to 4,000 lb of raw material. Formulate an LP that can be used to determine how Rylon can maximize profits.

Decision Variables: X_b = oz of Regular Brute sold
X_{eb} = oz of Luxury Brute sold
X_c = oz of Regular Chanelle sold
X_{ec} = oz of Luxury Chanelle sold
X_r = lbs of raw materials

Constraints:

(Labor) $3X_{eb} + 2X_{ec} + X_r \leq 6000$

(Raw Materials) $X_r \leq 4000$

(Production Caps) $X_b + X_{eb} - 3X_r \leq 0$

$X_c + X_{ec} - 4X_r \leq 0$

Obj Fun: Maximize Profits

Z = (Sales Revenue) − (Processing Costs) − (Raw Material Costs)

$\Rightarrow Z = (7X_b + 6X_c + 18X_{eb} + 14X_{ec}) - (4X_{eb} + 4X_{ec}) - (3X_r)$

$\Rightarrow Z = 7X_b + 6X_c + 14X_{eb} + 10X_{ec} - 3X_r$

(Production Caps) $X_b + X_{eb} = 3X_r$

$X_c + X_{ec} = 4X_r$

Solution: From Excel:

$X_b = 11333$ oz, $X_{eb} = 666.7$ oz, $X_c = 16000$ oz, $X_{ec} = 0$ oz

$X_r = 4000$ lbs

Max Z = \$172,666.67

2.] More Perfume: Suppose in the previous problem, 1 lb of raw material could be used to produce either 3 oz of Brute *or* 4 oz of Chanelle. How would this change the formulation.

Decision Variables: Add in the following:

X_{rb} = lbs of raw materials to convert to Regular Brute

X_{rc} = lbs of raw materials to convert to Regular Chanelle

(Production Cap Constraints) $X_b + X_{eb} = 3X_{rb}$

$X_c + X_{rc} = 4X_{rc}$

$X_{rc} + X_{rb} = X_r$

Solution: From Excel:

$X_b = 0$ oz, $X_{eb} = 666.7$ oz, $X_c = 15111$ oz

$X_{ec} = 0$ oz, $X_{rb} = 222.2$ lbs, $X_{rc} = 3777.8$ lbs

Max Z = \$88,000

3.] Chemical Production: Chemco produces three products: 1, 2, and 3. Each lb of raw material costs \$25. It undergoes processing and yields 3 oz of product 1 and 1 oz of product 2. It costs \$1 and takes 2 hrs of labor to process each pound of raw material. Each ounce of product 1 can be used in one of three ways:

- It can be sold for \$10/oz.
- It can be processed into 1 oz of product 2, which requires 2 hours of labor and costs \$1.
- It can be processed into 1 oz of product 3, which requires 3 hours of labor and costs \$2.

Each ounce of product 2 can be used in one of two ways:

- It can be sold for \$20/oz.
- It can be processed into 1 oz of product 3, which requires 1 hour of labor and costs \$6.

Product 3 is sold for \$30/oz. The maximum number of ounces of each product is 5,000 oz, 5,000 oz, and 3,000 oz, respectively. A maximum number of 25,000 labor hrs are available. Determine how Chemco can maximize profit.

Decision Variables: X_r = lbs of raw material purchased
X_{ij} = oz of product i that is processed into product j (X_{12}, X_{13}, X_{23})
X_{is} = oz of product i that is sold (X_{1s}, X_{2s}, X_{3s})
X_{ip} = oz of product i that is produced (X_{1p}, X_{2p})

Objective function: Maximize profits, profit = (Sales Revenue) − (Processing Costs) − (Purchasing Costs)

$$Z = (10X_{1s} + 20X_{2s} + 32X_{3s}) - (X_r + X_{12} + 2X_{13} + 6X_{23}) - (25X_r)$$

$$\Rightarrow Z = 10X_{1s} + 20X_{2s} + 32X_{3s} - X_{12} - 2X_{13} - 6X_{23} - 26X_r$$

Constraints:

(Labor) $2X_r + 2X_{12} + 3X_{13} + X_{23} \le 25000$

(Max Quantity)
$X_{1s} \le 5000$
$X_{2s} \le 5000$
$X_{3s} \le 3000$

(Processing Limits)
$X_{1s} \le X_{1p} - X_{12} - X_{13}$
$X_{2s} \le X_{2p} + X_{12} - X_{23}$
$X_{3s} \le .75X_{13} + .70X_{23}$

(Production)
$X_{1p} = 3X_r$
$X_{2p} = X_r$

note: we can eliminate the variable X_{1p} & X_{2p} by substituting them into the processing constraints

$$\Rightarrow \begin{array}{rrrrr} & & 2X_{12} + 3X_{13} + X_{23} + X_r & \le & 25000 \\ X_{1s} & & & \le & 5000 \\ & X_{2s} & & \le & 5000 \\ & & X_{3s} & \le & 3000 \\ X_{1s} & + X_{12} + X_{13} & - 3X_r & \le & 0 \\ X_{2s} & - X_{12} & + X_{23} - X_r & \le & 0 \\ X_{3s} & -.75X_{13} - .7X_{23} & & \le & 0 \end{array}$$

Solution: From Excel:
$X_{1s} = 5000$ oz, $X_{2s} = 3000$ oz, $X_{3s} = 3000$ oz, $X_{12} = 1500$ oz,
$X_{13} = 4000$ oz, $X_{23} = 0$ oz, $X_r = 3500$ lbs
max $Z = \$145{,}500$