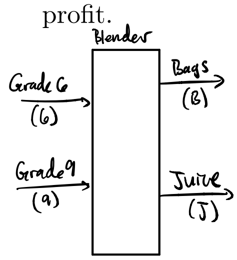
§3.8: Blending Problems

1.] Juicy Juice: O.J. Juice Company sells bags of oranges and cartons of orange juice. O.J. grades oranges on a scale of 1 (poor) to 10 (excellent). O.J. now has on hand 100,000 lbs of grade 9 oranges and 120,000 lbs of grade 6 oranges. The average quality of oranges sold in bags must be at least 7, and the average quality of the oranges used to produce orange juice must be at least 8. Each pound of oranges that is used for juice yields a revenue of \$1.50 and incurs a variable cost of \$1.05. Each pound of oranges sold in bags yields a revenue of \$0.50 and incurs a variable cost of \$0.20. Formulate an LP to help O.J. optimize profit.



Decision Variables:

X_{6B} = lbs of Grade 6 oranges used to make bags
X_{9B} = lbs of Grade 9 oranges used to make bags
X_{6J} = lbs of Grade 6 oranges used to make juice
X_{9J} = lbs of Grade 9 oranges used to make juice

Constraints:

(Available Quantities)
Grade 6: $X_{6B} + X_{6J} \le 120{,}000$
Grade 9: $X_{9B} + X_{9J} \le 100{,}000$

(Specifications)
Bags: $\dfrac{6X_{6B} + 9X_{9B}}{X_{6B} + X_{9B}} \ge 7 \Rightarrow -X_{6B} + 2X_{9B} \ge 0$

Juice: $\dfrac{6X_{6J} + 9X_{9J}}{X_{6J} + X_{9J}} \ge 8 \Rightarrow -2X_{6J} + X_{9J} \ge 0$

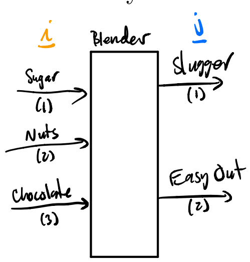
Obj Fun: Maximize Profit

Profit from Bags: $0.50 - 0.20 = 0.30/\text{lb}$
Profit from Juice: $1.50 - 1.05 = 0.45/\text{lb}$

$Z = 0.30(X_{6B} + X_{9B}) + 0.45(X_{6J} + X_{9J})$

Solution: From Excel: Max $Z = \$87{,}000$
$X_{6B} = 53{,}333$, $X_{9B} = 26{,}667$, $X_{6J} = 46{,}667$, $X_{9J} = 93{,}333$

2.] Candy Shop: You have decided to enter the candy business. You are considering producing two types of candies: Slugger Candy and Easy Out Candy, both of which consist solely of sugar, nuts, and chocolate. At present, you have in stock 100 oz of sugar, 20 oz of nuts, and 30 oz of chocolate. The mixture used to make Easy Out Candy must contain at least 20% nuts. The mixture used to make Slugger Candy must contain at least 10% nuts and 10% chocolate. Each ounce of Easy Out Candy can be sold for \$0.25 and each ounce of Slugger Candy for \$0.20. Formulate an LP that will enable you to maximize your revenue from candy sales.



Decision Variables: X_{ij} = ounces of ingredient i used to make candy j

$i = 1,2,3$ (Sugar, Nuts, Chocolate)
$j = 1,2$ (Slugger, Easy Out)

Constraints:

(Available Quantities)
Sugar: $X_{11} + X_{12} \le 100$
Nuts: $X_{21} + X_{22} \le 20$
Chocolate: $X_{31} + X_{32} \le 30$

(Specifications)
Easy Out Nut Prop: $\dfrac{X_{22}}{X_{12} + X_{22} + X_{32}} \ge .20 \Rightarrow -.20X_{12} + .80X_{22} - .20X_{32} \ge 0$

Slugger Nut Prop: $\dfrac{X_{21}}{X_{11} + X_{21} + X_{31}} \ge .10 \Rightarrow -.10X_{11} + .90X_{21} - .10X_{31} \ge 0$

Slugger Choco Prop: $\dfrac{X_{31}}{X_{11} + X_{21} + X_{31}} \ge .10 \Rightarrow -.10X_{11} - .10X_{21} + .90X_{31} \ge 0$

Obj Fun: Maximize Revenue

Slugger Revenue: $\$.20/\text{oz}$
Easy Out Revenue: $\$.25/\text{oz}$

$Z = .20(X_{11} + X_{21} + X_{31}) + .25(X_{12} + X_{22} + X_{32})$

Solution: From Excel: Max $Z = \$32.50$

$X_{11} = 60$, $X_{12} = 40$, $X_{21} = 10$, $X_{22} = 10$, $X_{31} = 30$, $X_{32} = 0$

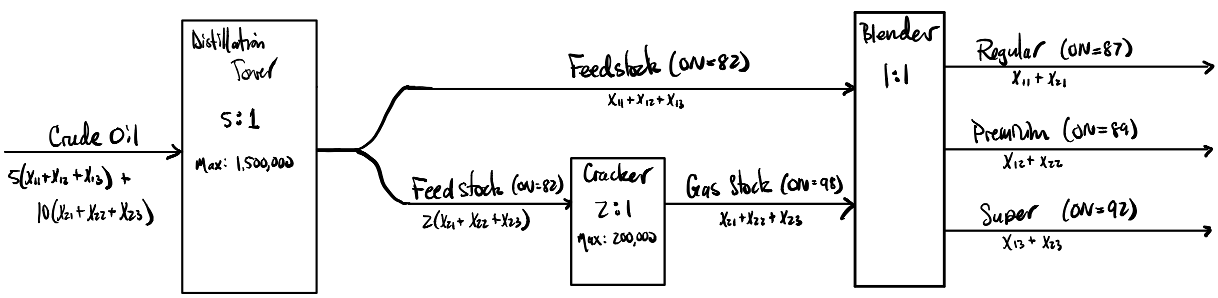
3.] Oil Refining: Shale Oil, located on the island of Aruba, has a capacity of 1,500,000 bbl of crude oil per day. The final products from the refinery include three types of unleaded gasoline with different octane numbers (ON): regular with ON = 87, premium with ON = 89, and super with ON = 92. The refining process encompasses three stages:

1.) a distillation tower that produces feedstock (ON = 82) at the rate of .2 bbl per bbl of crude oil,

2.) a cracker unit that produces gasoline stock (ON = 98) by using a portion of the feedstock produced from the distillation tower at the rate of .5 bbl per bbl of crude oil,

3.) and a blender unit that blends the gasoline stock from the cracker unit and the feedstock from the distillation tower.

The company estimates the net profit per barrel of the three types of gasoline to be \$6.70, \$7.20, and \$8.10, respectively. The input capacity of the cracker unit is 200,000 bbl of feedstock per day. The demand limits for regular, premium, and super gasoline are 50,000, 30,000, and 40,000 bbl, respectively, per day. Develop a model for determining the optimum production schedule for the refinery.



Decision Variables: x_{ij} = bbl/day of input stream i used to produce gasoline type j

$i = 1,2$ (feedstock from crude, gas stock from cracker)

$j = 1,2,3$ (Regular, Premium, Super)

Obj Fun: Maximize Profit: $z = 6.70(x_{11}+x_{21}) + 7.20(x_{12}+x_{22}) + 8.10(x_{13}+x_{23})$

Totals: Map out the amounts flowing in/out of each phase of blending/refining in terms of the variables

- Output: Regular $= x_{11}+x_{21}$, Premium $= x_{12}+x_{22}$, Super $= x_{13}+x_{23}$
- Input into blender: Feedstock $= x_{11}+x_{12}+x_{13}$, Gas Stock $= x_{21}+x_{22}+x_{23}$
- Input into cracker: Feedstock $= 2(x_{21}+x_{22}+x_{23})$ because the output is $x_{21}+x_{22}+x_{23}$, there must been exactly twice that amount going in since the rate of cracking is 2:1.
- Distillation output: Feedstock $= (x_{11}+x_{12}+x_{13}) + 2(x_{21}+x_{22}+x_{23})$ because the feedstock is split into that entering blender and that entering cracker.
- Input into Distillation: Crude Oil $= 5(x_{11}+x_{12}+x_{13}) + 10(x_{21}+x_{22}+x_{23})$ because the distiller rate is 5:1

Average Octane Numbers: The octane number of a gasoline product is the weighted average of the octane numbers of the input streams used in blending.

(Regular Octane Specification) $$\frac{82X_{11} + 98X_{21}}{X_{11} + X_{21}} \geq 87 \quad \Rightarrow \quad -5X_{11} + 11X_{21} \geq 0$$

(Premium Octane Specification) $$\frac{82X_{12} + 98X_{22}}{X_{12} + X_{22}} \geq 89 \quad \Rightarrow \quad -7X_{12} + 9X_{22} \geq 0$$

(Super Octane Specification) $$\frac{82X_{13} + 98X_{23}}{X_{13} + X_{23}} \geq 92 \quad \Rightarrow \quad -10X_{13} + 6X_{23} \geq 0$$

Constraints:

(Crude oil Capacity) $5X_{11} + 5X_{12} + 5X_{13} + 10X_{21} + 10X_{22} + 10X_{23} \leq 1{,}500{,}000$

(Cracker Capacity) $2X_{21} + 2X_{22} + 2X_{23} \leq 200{,}000$

(Regular Demand) $X_{11} + X_{21} \leq 50{,}000$

(Premium Demand) $X_{12} + X_{22} \leq 30{,}000$

(Super Demand) $X_{13} + X_{23} \leq 40{,}000$

(Regular ON) $-5X_{11} + 11X_{21} \geq 0$

(Regular ON) $-7X_{12} + 9X_{22} \geq 0$

(Regular ON) $-10X_{13} + 6X_{23} \geq 0$

Solutions: From Excel: Max $Z = \$875{,}000$

$X_{11} = 34{,}375$, $X_{12} = 16{,}875$, $X_{13} = 15{,}000$, $X_{21} = 15{,}625$, $X_{22} = 13{,}125$, $X_{23} = 25{,}000$