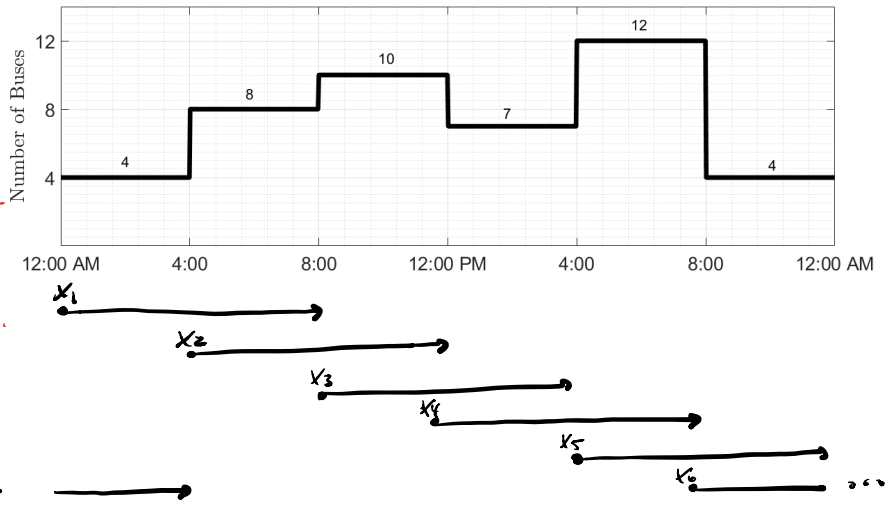
§3.5: Scheduling Problems

1.] Bus Scheduling: Progress City is studying the feasibility of introducing a mass-transit bus system to reduce in-city driving. The study seeks the minimum number of buses that can handle the transportation needs. After gathering necessary information, the city engineer noticed that the minimum number of buses needed fluctuated with time of the day, and that the required number of buses could be approximated by constant values over successive 4-hr intervals. The figure below summarizes the engineer's findings. To carry out the required daily maintenance, each bus can operate only 8 successive hours a day.



Note: - Divisibility Assumption is absolutely violated, i.e. cannot have fractions of buses. Need to round at the end or reformulate as integer problem.
- Static Problem - same schedule each week

Decision Variables: x_1 = # buses starting shift at 12:01 AM
x_2 = # buses starting shift at 4:01 AM
⋮
x_6 = # buses starting shift at 8:00 PM

Pro-Tip: For scheduling problems, do not assume the shift. Let the model determine the shifts by defining the decision variables as "# of workers starting at time t" instead of "# of workers between t_1 & t_2"

Constraints: (8 hr shift with min # of buses)

(# of buses in rotation.)

12:01–4:00 AM:	$x_1 + x_6$	≥ 4
4:01–8:00 :	$x_1 + x_2$	≥ 8
8:01–12:00 PM:	$x_2 + x_3$	≥ 10
12:01–4:00 :	$x_3 + x_4$	≥ 7
4:01–8:00 :	$x_4 + x_5$	≥ 12
8:01–12:00 AM:	$x_5 + x_6$	≥ 4

$x_1, x_2, x_3, x_4, x_5, x_6 \geq 0$

Obj Fun: Minimize total # of buses

$Z = x_1 + x_2 + x_3 + x_4 + x_5 + x_6$

Note: Model can be modified for consideration of shift lengths.

Solution: From Excel:

$x_1 = 4, x_2 = 4, x_3 = 6, x_4 = 8, x_5 = 4, x_6 = 0$

$\min Z = 26$

2.] On most university campuses, students are contracted by academic departments to perform office errands. The need for such service fluctuates during the work hours (8:00 AM to 5:00 AM). In the math department, the minimum number of students needed is 2 between 8:00 AM and 10:00 AM, 3 between 10:01 AM and 11:00 AM, 4 between 11:01 AM and 1:00 PM, and 3 between 1:01 PM and 5:00 PM. Each student is allotted 3 consecutive hours (except those starting at 3:01 PM, who work for 2 hr, and those who start at 4:01 PM, who work for 1 hr). Because of their flexible schedule, students can usually start at any hour during the workday, except at lunchtime (12:00 noon). Develop the LP model.

Data:

	slot	min students
(1)	8:01 – 9:00 AM	2
(2)	9:01 – 10:00	2
(3)	10:01 – 11:00	3
(4)	11:01 – 12:00 PM	4
(5)	12:01 – 1:00	4
(6)	1:01 – 2:00	3
(7)	2:01 – 3:00	3
(8)	3:01 – 4:00	3
(9)	4:01 – 5:00	3

Decision Variables:

x_1 = # of students starting at 8:01 AM

x_2 = # of students starting at 9:01 AM

⋮ ⋮ ⋮

x_9 = # of students starting at 4:01 PM

Constraints: (min # of students in each slot for 3 hr shifts, except at 3:01 and 4:01)

(# of students working)

8:01 – 9:00 AM: $x_1 \geq 2$

9:01 – 10:00: $x_1 + x_2 \geq 2$

10:01 – 11:00: $x_1 + x_2 + x_3 \geq 3$

11:01 – 12:00 PM: $x_2 + x_3 + x_4 \geq 4$

12:01 – 1:00: $x_3 + x_4 \geq 4$

1:01 – 2:00: $x_4 + x_6 \geq 3$

2:01 – 3:00: $x_6 + x_7 \geq 3$

3:01 – 4:00: $x_6 + x_7 + x_8 \geq 3$

4:01 – 5:00: $x_7 + x_8 + x_9 \geq 3$

(no one starts at noon) $x_5 = 0$

$x_1, x_2, x_3, x_4, x_6, x_7, x_8, x_9 \geq 0$

Obj. Fun: Minimize # of student workers

$$Z = x_1 + x_2 + x_3 + x_4 + x_6 + x_7 + x_8 + x_9.$$

Solution: From Excel:

$x_1 = 2, x_3 = 1, x_4 = 3, x_7 = 3$

$x_2 = x_5 = x_6 = x_8 = x_9 = 0$

$\min Z = 9$

Note: Divisibility Assumption is violated again!