§3.4: Cost Minimization Problems

1.] Diet Problem: A diet requires that food intake comes from one of the four basic food groups: chocolate cake, ice cream, soda, and cheesecake. At present, the following four foods are available for consumption: brownies, chocolate ice cream, Mountain Dew, and strawberry cheesecake. Each brownie costs \$0.75, each scoop of ice cream costs \$1.10, each can of Mountain Dew costs \$1.50, and each piece of strawberry cheesecake costs \$3.50. Each day, the diet requires at least 500 calories, 6 oz of chocolate, 10 oz of sugar, and 8 oz of fat. The nutritional content per unit of each food is shown in the following table. Formulate the LP problem that can be used to satisfy the daily nutritional requirements at minimum cost.

Type of Food	Calories	Chocolate (ounces)	Sugar (ounces)	Fat (ounces)
Brownie	400	3	2	2
Chocolate Ice Cream (1 scoop)	200	2	2	4
Mountain Dew (1 can)	150	0	4	1
Cheesecake (1 piece)	500	0	4	5

Decision Variables: x_1 = # of Brownies
x_2 = # of scoops of ice cream
x_3 = cans of soda
x_4 = pieces of cheesecake.

Note: • Divisibility Assumption – our answer may end up being non-integer, ie half a brownie or 3/4 can of soda.
• Certainty Assumption – nutritional facts can be found on food labels

Constraints:

(Calorie Intake) $400x_1 + 200x_2 + 150x_3 + 500x_4 \geq 500$

(Choc Requirement) $3x_1 + 2x_2 \geq 6$

(Sugar Requirement) $2x_1 + 2x_2 + 4x_3 + 4x_4 \geq 10$

(Fat Requirement) $2x_1 + 4x_2 + x_3 + 5x_4 \geq 8$

$x_1, x_2, x_3, x_4 \geq 0$

Objective function: Minimize Cost in cents, not dollars

$$z = 75x_1 + 110x_2 + 150x_3 + 350x_4$$

Historical Note: A diet problem was one of the first LPs to be solved by computer.

2.] HEART VALVES: U.S. Labs manufactures mechanical heart valves from the heart valves of pigs. Different heart operations require valves of different sizes. U.S. Labs purchases pig valves from three different suppliers. The cost and size mix of the valves purchased from each supplier are given in the table below. Each month, U.S. Labs places one order with each supplier. At least 500 large, 300 medium, and 300 small valves must be purchased each month. Because of limited availability of pig valves, at most 700 valves per month can be purchased from each supplier. Formulate an LP that can be used to minimize the cost of acquiring the needed valves.

Supplier	Cost per valve ($)	Large (%)	Medium (%)	Small (%)
1	5	40	40	20
2	4	30	35	35
3	3	20	20	60

Decision Variables: x_1 = # valves from supplier 1
x_2 = # valves from supplier 2
x_3 = # valves from supplier 3

Note: Certainty Assumption may be violated as the % are likely estimated and size is probably subjective.

Constraints:

Size Constraints:

(Large) $.4x_1 + .3x_2 + .2x_3 \geq 500$

(medium) $.4x_1 + .35x_2 + .2x_3 \geq 300$

(small) $.20x_1 + .35x_2 + .6x_3 \geq 300$

Max Supplies Constraints:

(Supp 1) $x_1 \leq 700$

(Supp 2) $x_2 \leq 700$

(Supp 3) $x_3 \leq 700$

$x_1, x_2, x_3 \geq 0$

Obj. Fun: Minimize total cost

$Z = 5x_1 + 4x_2 + 3x_3$