§3.2 (part 1): Two Variable LP Models

1.] The Reddy Mikks Company: The Reddy Mikks company produces both interior and exterior paints from two raw materials, M_1 and M_2. The following table summarizes the basic data of the problem:

	Tons of raw material per ton of *Exterior Paint*	*Interior Paint*	Maximum daily availability (tons)
Raw Material, M_1	6	4	24
Raw Material, M_2	1	2	6
Profit per ton ($1000)	5	4	

The daily demand for interior paint cannot exceed that for exterior paint by more than 1 ton. Further, the maximum daily demand for interior paint is 2 tons. Reddy Mikks wishes to determine the optimal product mix of interior and exterior paints that maximizes daily profits. Formulate the mathematical model.

Objective: Maximize Profits
Constraints: stay with availibility. Don't exceed market demand.

Decision Variables: x_1 = tons of exterior paint produced
x_2 = tons of interior paint produced

Obj. Function: $Z = 5x_1 + 4x_2$ (in $1000s)

Constraints: Availibility Constraints:

(M_1) $6x_1 + 4x_2 \le 24$

(M_2) $x_1 + 2x_2 \le 6$

Demand Constraints:

$x_2 - x_1 \le 1$

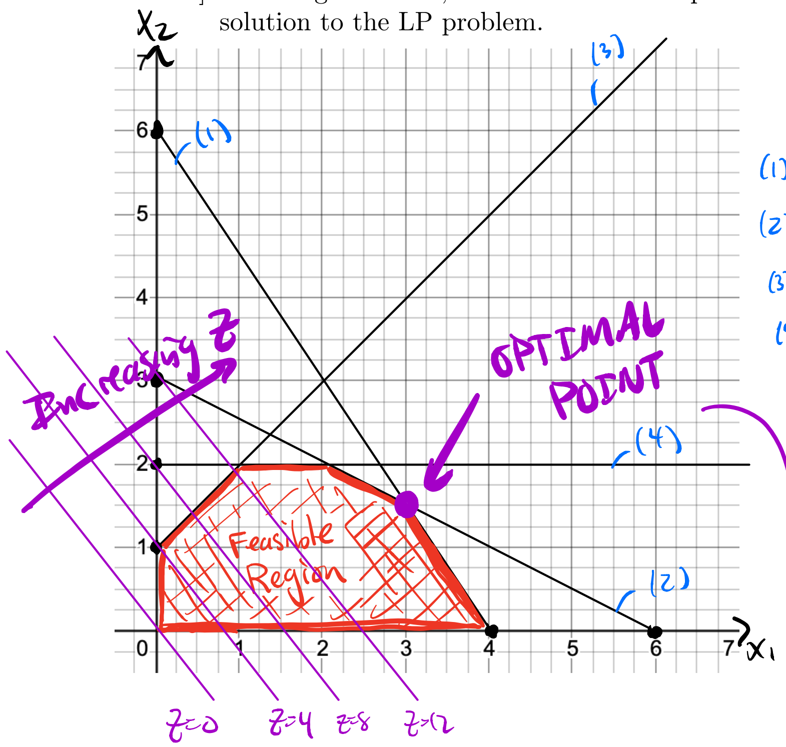
$x_2 \le 2$

Sign Restriction: $x_1 \ge 0$
$x_2 \ge 0$

Complete Formulation

$$\boxed{\begin{aligned} &\text{Max } Z = 5x_1 + 4x_2 \\ &\text{s.t.} \\ &6x_1 + 4x_2 \le 24 \\ &x_1 + 2x_2 \le 6 \\ &-x_1 + x_2 \le 1 \\ &x_2 \le 2 \\ &x_1, x_2 \ge 0 \end{aligned}}$$

2.] In the figure below, sketch the feasible space for the Reddy Mikks problem, and determine the optimal solution to the LP problem.



Constraints:

(1) $6x_1+4x_2 \le 24$ → $y = \frac{3}{2}x+6$ (0,6), (4,0)

(2) $x_1+2x_2 \le 6$ → $y = -\frac{1}{2}x+3$ (0,3), (6,0)

(3) $-x_1+x_2 \le 1$ → $y = x+1$ (0,1), (-1,0)

(4) $x_2 \le 2$ → $y=2$ (0,2)

$x_1, x_2 \ge 0$ → must be in Quad I

Lines: ($y=x_2$, $x=x_1$)

Intercepts:

The point in the feasible region that optimizes the obj. function is the corner point (3, 1.5).

$x_1 = 3$ tons of exterior paint
$x_2 = 1.5$ tons of interior paint

creates \$21,000 in profit.

Objective Function:

$z = 5x_1+4x_2$

Pick a few values for z:

$z=0 \to 0 = 5x_1+4x_2 \to x_2 = -\frac{5}{4}x_1$

$z=4 \to 4 = 5x_1+4x_2 \to x_2 = -\frac{5}{4}x_1+1$

$z=8 \to 8 = 5x_1+4x_2 \to x_2 = -\frac{5}{4}x+2$

3.] Is the point $x_1 = 2$ and $x_2 = 2$ a feasible solution? What is the daily profit? Determine the unused amounts of raw materials, M_1 and M_2.

- Yes, it is on the boundary corner point, but it is not optimal.
- Profit: $z = 5x_1+4x_2 = 5(2)+4(2) = 18 \to \boxed{z = \$18,000}$
- Unused M_1: $24-6x_1-4x_2 = 24-6(2)-4(2) = 24-20 = \boxed{4 \text{ tons unused}}$

Unused M_2:

$6-x_1-2x_2 = 6-2-2(2) = 0$

$\boxed{0 \text{ tons unused}}$

4.] Suppose that Reddy Mikks sells its exterior paint to a single wholesaler at a quantity discount. The profit per ton is \$5000 if the contractor buys no more than 2 tons daily and \$4500 otherwise. Express the objective function mathematically. Is the resulting function linear?

- The new objective function would be piecewise linear:

$$z = \begin{cases} 5x_1+4x_2 & \text{if } x_1 \le 2 \\ 4.5x_1+4x_2 & \text{if } x_1 > 2 \end{cases} \leftarrow \text{not linear}$$

Although the point (3, 1.5) is still optimal.