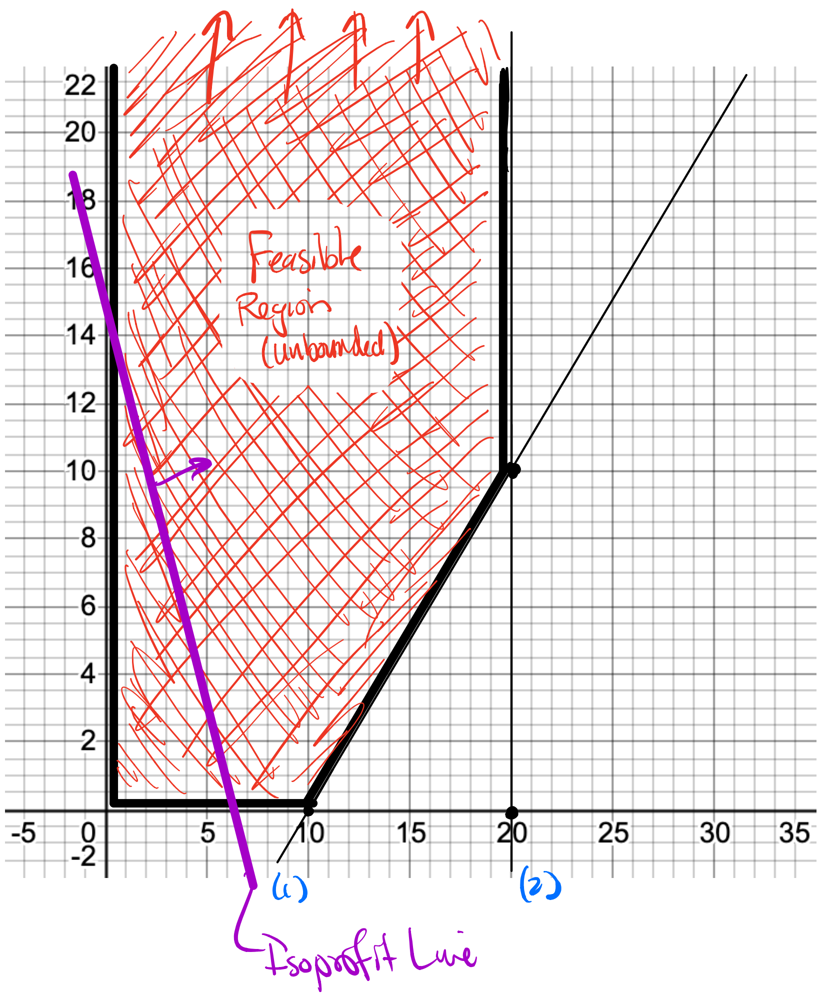
§3.3 (part 2): Unbounded and Infeasible Solutions

1.] Determine the optimal solution, if possible, to the LP below by sketching the feasible space on the graph provided.

$$\text{Maximize:} \quad z = 2x + y$$

$$\begin{aligned} \text{Subject to:} \quad x - y &\le 10 \quad (1) \\ 2x &\le 40 \quad (2) \\ x, y &\ge 0 \end{aligned}$$



Feasible Region

(1) $y \ge x - 10$

(2) $x \le 20$

Objective Function:

$y = -2x + z$ Slope: $m = -2$

Here, the feasible region is unbounded. In fact the y decision variable can be made arbitrarily large making the profit arbitrarily large as well. The constraints do not restrict the variables enough to produce a unique optimal solution.

Max Profit → can be made arbitrarily large by increasing y.

2.] Determine the optimal solution, if possible, to the LP below by sketching the feasible space on the graph provided.

$$\text{Maximize:} \qquad z = 3x + 2y$$

$$\begin{aligned} \text{Subject to:} \quad 2x + y &\leq 2 \quad (1) \\ 3x + 4y &\geq 12 \quad (2) \\ x, y &\geq 0 \end{aligned}$$

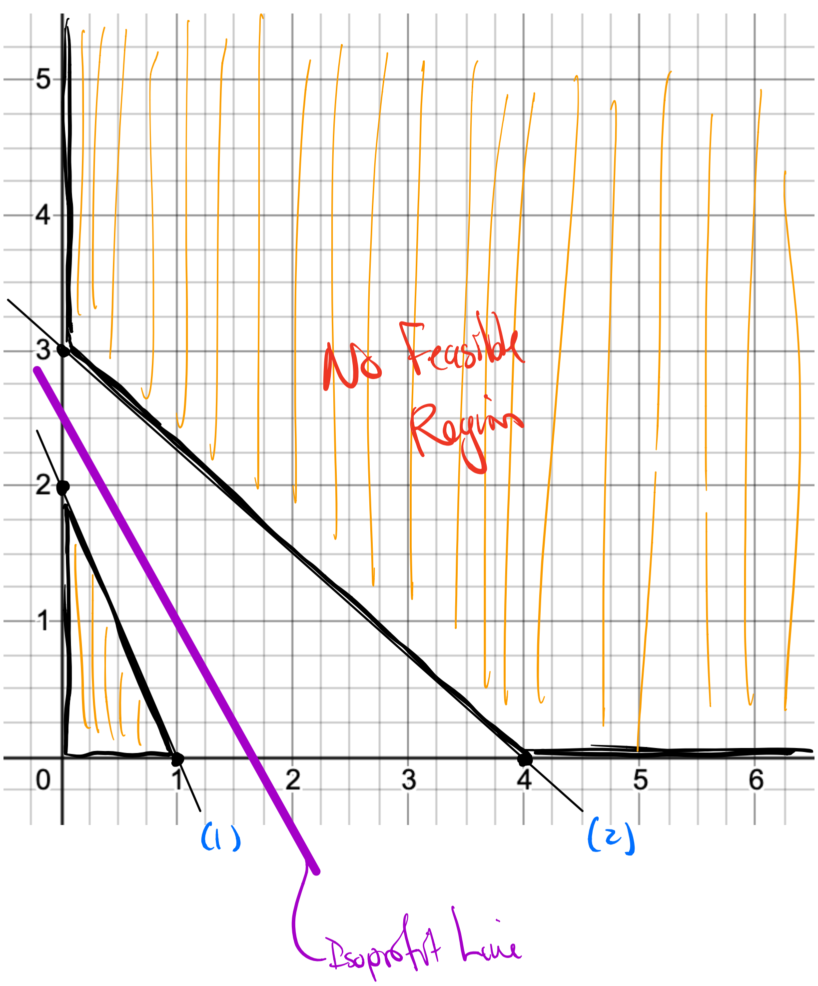
Feasible Region

(1) $y \leq -2x + 2$

(2) $y \geq -\frac{3}{4}x + 3$

Objective Function:

$y = -\frac{3}{2}x + \frac{z}{2}$ Slope: $m = -\frac{3}{2}$



Here, the two orange spaces have no intersection and hence the feasible region is empty. Therefore, there are no solutions that satisfy the constraints. The problem is infeasible.