§3.2 (part 2): Two Variable LP Models

1.] Show & Sell: Show & Sell can advertise its products on local radio and television (TV). The advertising budget is limited to \$10,000 a month. Each minute of radio advertising costs \$15, and each minute of TV commercials cost \$300. Show & Sell likes to advertise on radio at least twice as much as on TV. In the meantime, it is not practical to use more than 400 minutes of radio advertising a month. From past experience, advertising on TV is estimated to be 25 times as effective as on radio. Formulate the LP for Show & Sell.

Decision Variables: x_1 = mins on radio
x_2 = mins on TV

Budget Constraints: $15x_1 + 300x_2 \le 10{,}000$

other Constraints: $x_1 \ge 2x_2$
$x_1 \le 400$

Sign Restriction: $x_1, x_2 \ge 0$

Obj. Fun: Max effectiveness
$Z = x_1 + 25x_2$

LP Problem

Max $Z = x_1 + 25x_2$

S.T.

$15x_1 + 300x_2 \le 10{,}000$

$x_1 - 2x_2 \ge 0$

$x_2 \le 400$

$x_1, x_2 \ge 0$

2.] Day Trader: Day Trader wants to invest a sum of money that would generate an annual yield of at least \$10,000. Two stock groups are available: blue chips and high-tech, with average annual yields of 10% and 25%, respectively. Though high-tech stocks provide higher yield, they are more risky, and Trader wants to limit the amount invested in these stocks to no more than 60% of the total investment. What is the minimum amount Trader should invest in each stock group to accomplish the investment goal? Formulate the LP for Day Trader.

Decision Variables: x_1 = investment amount in blue chips
x_2 = investment amount in high-tech

Investment Goal (Constraint): $1.10x_1 + 1.25x_2 \ge 10{,}000$

Risk Constraint: $x_2 \le .6(x_1 + x_2)$

Sign Restriction: $x_1, x_2 \ge 0$

Obj. Fun: Minimize investment
$Z = x_1 + x_2$

LP – Problem

Min $Z = x_1 + x_2$

S.T.

$1.10x_1 + 1.25x_2 \ge 10{,}000$

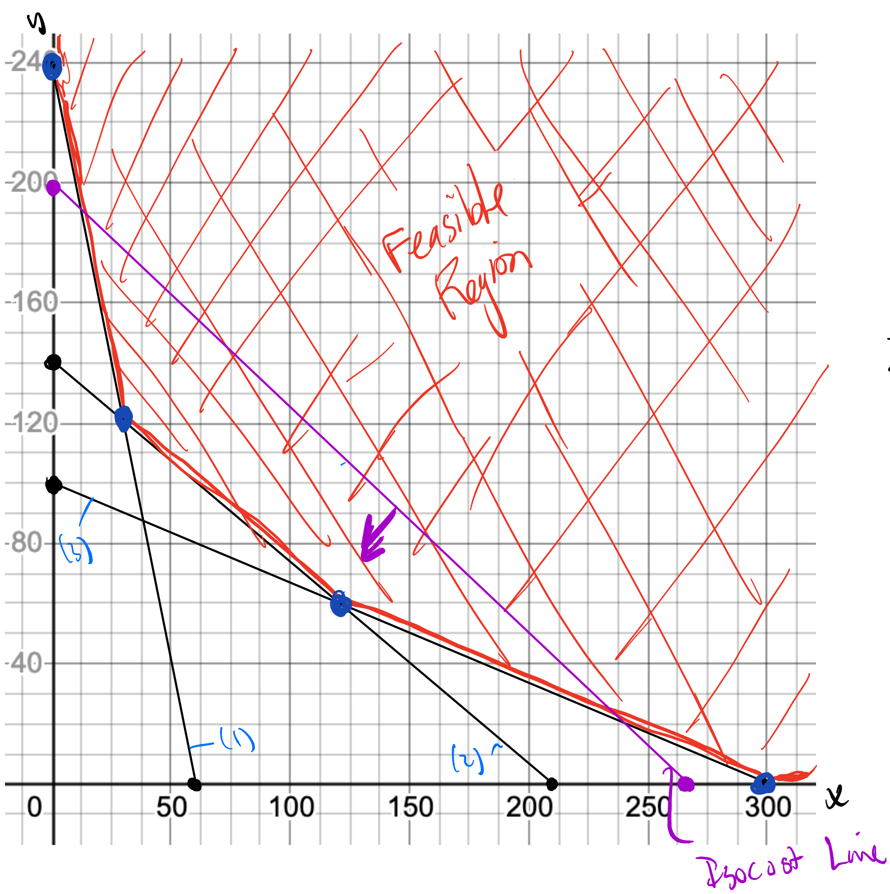
$.6x - .4x_2 \ge 0$

$x_1, x_2 \ge 0$

3.] Determine the optimal solution to the LP below by sketching the feasible space on the graph provided.

$$\text{Minimize:} \quad z = 6x + 8y$$

$$\text{Subject to:} \quad \begin{aligned} 40x + 10y &\geq 2400 \quad (1) \\ 10x + 15y &\geq 2100 \quad (2) \\ 5x + 15y &\geq 1500 \quad (3) \\ x, y &\geq 0 \end{aligned}$$



x- and y-ints

(1) (0,240) (60,0)

(2) (0,140) (210,0)

(3) (0,100) (300,0)

Isocost Line: $z = 6x + 8y$

$y = -\frac{3}{4}x + \frac{z}{8}$

$z = 1600 \Rightarrow y = -\frac{3}{4}x + 200$

(0,100) (266.6,0)

Optimal solution is either at the intersection of (1) and (2) or (2) and (3).

• (1) and (2): $\begin{cases} 40x + 10y = 2400 \\ 10x + 15y = 2100 \end{cases} \Rightarrow \begin{cases} 40x + 10y = 2400 \\ 40x + 60y = 8400 \end{cases} \Rightarrow \begin{cases} -50y = -6000 \\ 40x + 10y = 2400 \end{cases}$

$\Rightarrow \begin{cases} y = 120 \\ 40x + 1200 = 2400 \end{cases} \Rightarrow \begin{cases} y = 120 \\ x = 30 \end{cases} \Rightarrow z = 6(30) + 8(120) = \boxed{1140}$

Optimal Solution $\boxed{x = 30 \\ y = 120}$

• (2) & (3) $\Rightarrow \begin{cases} 10x + 15y = 2100 \\ 5x + 15y = 1500 \end{cases} \Rightarrow \begin{cases} 5x = 600 \\ 5x + 15y = 1500 \end{cases} \Rightarrow \begin{cases} x = 120 \\ 600 + 15y = 1500 \end{cases} \Rightarrow \begin{cases} x = 120 \\ y = 60 \end{cases}$

$z = 6(120) + 8(60) = 1200$