§3.6: Investment Problems

1.] **Net Present Value:** Suppose there are two investments with varying cashflows as shown in the table below. That is, to invest in Investment 1, \$10,000 is needed to invest now, a cash inflow of \$24,000 occurs one year from now, and we owe another \$14,000 two years from now. Compute the net present value (NPV) of each investment if the annual return is assumed to be $r = 0.20$. Which investment is better? What if $r = 0.02$?

	Cash Flow (\$)		
Investment	*Year 0*	*Year 1*	*Year 2*
1	−10000	24000	−14000
2	−6000	8000	−1000

Net Cash Flow (assuming dollar amount is constant each year (bad assumption))

Inv_1: $-10000 + 24000 - 14000 = 0$

Inv_2: $-6000 + 8000 - 1000 = 1000$ ← Seems better

$r = .20$: $NPV_1 = -10000 + \frac{24000}{(1+.20)} - \frac{14000}{(1+.20)^2} = 27.78$ ← Better Return

$NPV_2 = -6000 + \frac{8000}{(1+.20)} - \frac{1000}{(1+.20)^2} = -27.78$

$r = .02$: $NPV_1 = -10000 + \frac{24000}{(1+.02)} - \frac{14000}{(1+.02)^2} = 73.05$

$NPV_2 = -6000 + \frac{8000}{(1+.02)} - \frac{1000}{(1+.02)^2} = 881.97$ ← Better Return

2.] **Capital Budgeting:** Two investments with varying cashflows are available, as shown in the table below. Suppose at time 0, a \$10,000 is available to invest, and at time 1, \$7,000 is available to invest. Assuming $r = 0.10$, set up an LP whose solution maximizes the NPV obtained from these investments. Assume that any fraction of an investment can be purchased.

	Cash Flow (\$)			
Investment	*Year 0*	*Year 1*	*Year 2*	*Year 3*
1	−6000	−5000	7000	9000
2	−8000	−3000	9000	7000

Decision Variables: x_1 = fraction to be invested in Investment 1
x_2 = fraction invested in Investment 2

Constraints:

(time 0 max) $6000x_1 + 8000x_2 \le 10000$

(time 1 max) $5000x_1 + 3000x_2 \le 7000$

(Fraction) $x_1 \le 1$

(Fraction) $x_2 \le 1$

$x_1, x_2 \ge 0$

Note: Although x_1 and x_2 are both fractions, they need not add to 1. We're deciding if it is desirable to go "all-in" on an investment or not.

Obj. Fun: Maximize Combined NPV: $Z = 2001.50x_1 + 1969.95x_2$

$NPV_1 = -6000 + \frac{-5000}{+.10} + \frac{7000}{(1+.10)^2} + \frac{9000}{(1+.10)^3} = 2001.50$

$NPV_2 = -8000 + \frac{-3000}{1+.10} + \frac{9000}{(1+.10)^2} + \frac{7000}{(1+.10)^3} = 1969.95$

Solution: From Excel $x_1 = 0.5$, $x_2 = 1$, Max $Z = \$2970.70$

3.] Investment Strategy: Fox Enterprises is considering six projects for possible construction over the next four years, with cashflows occurring at the end of the year. Fox can undertake any of the projects partially or completely. A partial undertaking of a project will prorate both the return and cash outlays proportionately. The NPV on returns and cash outlays for the projects are given in the table below.

	Cash Outlay ($1000)				
Project	*Year 1*	*Year 2*	*Year 3*	*Year 4*	Return ($1000)
1	10.5	14.4	2.2	2.4	32.40
2	8.3	12.6	9.5	3.1	35.80
3	10.2	14.2	5.6	4.2	17.75
4	7.2	10.5	7.5	5.0	14.80
5	12.3	10.1	8.3	6.3	18.20
6	9.2	7.8	6.9	5.1	12.35
Available Funds ($1000)	60.0	70.0	35.0	20.0	

Suppose that any funds left at the end of the year are used in the next year. Formulate the LP that maximizes total return.

Decision Variables: x_i = partial or full undertaking of project i, $i = 1,2,\dots,6$.
y_i = amount leftover from year i, $i = 1,2,3$

Constraints:

(Available funds in year i)

$$10.5x_1 + 8.3x_2 + 10.2x_3 + 7.2x_4 + 12.3x_5 + 9.2x_6 \le 60$$
$$14.4x_1 + 12.6x_2 + 14.2x_3 + 10.5x_4 + 10.1x_5 + 7.8x_6 - y_1 \le 70$$
$$2.2x_1 + 9.5x_2 + 5.6x_3 + 7.5x_4 + 8.3x_5 + 6.9x_6 - y_2 \le 35$$
$$2.4x_1 + 3.1x_2 + 4.2x_3 + 5.0x_4 + 6.3x_5 + 5.1x_6 - y_3 \le 20$$

(Leftovers in year i)

$$10.5x_1 + 8.3x_2 + 10.2x_3 + 7.2x_4 + 12.3x_5 + 9.2x_6 + y_1 = 60$$
$$14.4x_1 + 12.6x_2 + 14.2x_3 + 10.5x_4 + 10.1x_5 + 7.8x_6 - y_1 + y_2 = 70$$
$$2.2x_1 + 9.5x_2 + 5.6x_3 + 7.5x_4 + 8.3x_5 + 6.9x_6 - y_2 + y_3 = 35$$

(Fractional/non-negative investments)

$$x_1, x_2, x_3, x_4, x_5, x_6 \le 1$$
$$x_1, x_2, x_3, x_4, x_5, x_6 \ge 0$$

Obj Fun: Maximize NPV on returns.

$$Z = 32.4x_1 + 35.80x_2 + 17.75x_3 + 14.80x_4 + 18.20x_5 + 12.35x_6$$

Solution: From Excel: $x_1 = x_2 = x_3 = x_4 = x_5 = 1$, $x_6 = .71$
$y_1 = 4.96$, $y_2 = 7.62$, $y_3 = 4.62$

Max $Z = \$127,722.76$