

## CHAPTER 1: MODEL BUILDING

- 1.] **TICKET PURCHASING PROBLEM:** A businessperson has a 5-week commitment traveling from Fayetteville (FYV) and Denver (DEN). Weekly departure from Fayetteville occurs on Mondays for return on Wednesdays. A regular roundtrip ticket costs \$400, but a 20% discount is granted if the roundtrip dates span a weekend. A one-way ticket in either direction costs 75% of the regular price. How should the tickets be bought for the 5-week trip?

Objective: Minimize Cost of the trip.

Constraints: FYV  $\rightarrow$  DEN must occur on Monday  
DEN  $\rightarrow$  FYV must occur on Wednesday

In this problem, we can construct and list a few feasible solutions. Which one yields the minimum cost?

Sol 1: Buy 5 Round trips, FYV-DEN-FYV, for departure on Mon and return on wed of the same week.

$$\text{Cost: } 5 \cdot (\$400) = \$2000$$

Sol 2: Buy one FYV-DEN, four DEN-FYV-DEN that span weekends, and one DEN-FYV.

$$\text{Cost: } .75(\$400) + 4(.8)(\$400) + .75(\$400) = \$1880$$

Sol 3: Buy one FYV-DEN-FYV with departure in first week and return in last week. Buy four other DEN-FYV-DEN spanning weekends.

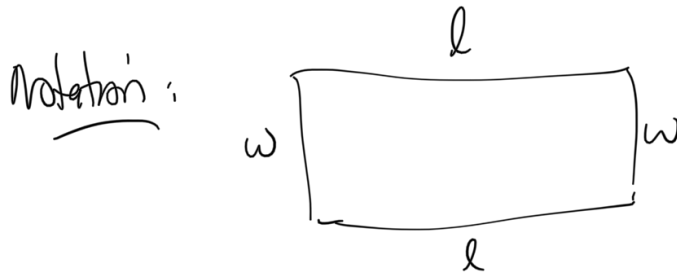
$$\text{Cost: } 5(.8)(\$400) = \$1600$$

Optimal solution.

- 2.] GARDEN PROBLEM: A home owner is in the process of building a backyard vegetable garden. The garden must take on a rectangular shape to facilitate row irrigation. To keep critters out, the garden must be fenced. The owner has enough material to build a fence of length  $L = 100$  ft. The goal is to fence in the largest possible area.

Objective: Maximize Area

Constraint: Use only the amount of fencing available.



Obj. Function:  $Z = lw$

Constraints:  $2w + 2l = 100$

$$l \geq 0$$

$$w \geq 0$$

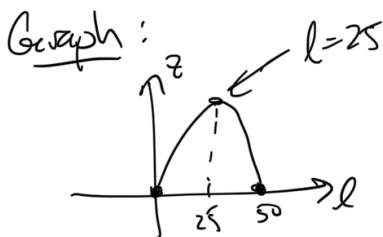
} Sign Restriction  
or non-negativity constraints

• Use the constraint to eliminate a variable in the obj. function:

$$\bullet \quad 2w + 2l = 100 \Rightarrow w + l = 50 \Rightarrow w = 50 - l$$

$$\bullet \quad Z = lw \Rightarrow Z = l(50 - l) \Rightarrow Z(l) = 50l - l^2$$

} maximize this function.



Calculus:

$$Z'(l) = 50 - 2l$$

$$0 = 50 - 2l$$

$$\boxed{l = 25} \Rightarrow \boxed{w = 25}$$

Pre-calculus:

Parabola, max occurs at

$$l = -\frac{b}{2a} = -\frac{50}{2(-1)} = 25$$